Engineering Failure Analysis 18 (2011) 117–123

Contents lists available at ScienceDirect

Engineering Failure Analysis

journal homepage: www.elsevier.com/locate/engfailanal

Optimisation of a beam in bending subjected to severe inertia impact loading

José María Mínguez [a,*], Jeffrey Vogwell [b]

[a] Dpto. de Física Aplicada II, Facultad de Ciencia y Tecnología, Universidad de Bilbao, Aptdo. 644, 48080 Bilbao, Spain
[b] Department of Mechanical Engineering, Faculty of Engineering and Design, University of Bath, UK

ARTICLE INFO

Article history:
Received 19 August 2010
Accepted 19 August 2010
Available online 27 August 2010

Keywords:
Impact failure
Braking system
Beam bending energy

ABSTRACT

This paper summarises an optimal analytical study of a bar when it is subjected to shock inertia loading such that the severe bending causes permanent yielding. The findings are related to a practical case study application and help explain the necessary actions that have to be taken to minimise the impact damage. The analysis relates the sudden strain energy which the bar gains following an impact (when either the stationary bar is collected by a body moving with substantial momentum or alternatively when the moving bar is suddenly brought to a halt) and establishes the bending strength as a function of mass, stiffness and other kinematic conditions. The optimal analysis results in a surprising paradox which is borne out in the actual detail design solution to a critical component which had caused major maintenance problems in the braking system of a railed vehicle. The study reveals some interesting findings relating to the best bar configuration in terms of geometrical design, support locations and material choice.

1. Introduction

In this work we study a bar which is subjected to a severe impact causing bending and hence determine the subsequent shock stresses that occur due to the inertia beam force created. The objective of the analysis is to determine the optimal arrangement as although maximising bending strength and stiffness is necessary, the adding of material increases mass and thus the magnitude of the inertia force acting. Parameters which have been considered are the bar material properties, its geometry and the spacing position of the stops or, alternatively, the grabbing supports.

Two cases are considered, firstly when the bar is initially lying in a stationary state and then collected by a pair of striking grips which are moving at a speed V and with considerable momentum thus producing action forces F and causing the bar to move suddenly to that speed as shown in Fig. 1a. Also, the reverse situation, shown in Fig. 1b, which is where the beam moves with speed V and is then suddenly stopped by a pair of equally spaced rigid stops producing reaction forces F. Such situations occur in practise, in the braking system of the bobsleigh start track of the University of Bath (UK) [1], which is what stimulated the need for the study.

2. Strain energy

It is assumed that the bar of mass m is collected from stationary by a heavy vehicle of mass M ($M \gg m$), which moves at speed V_1 and forces the bar to move at speed V by forces F (see Fig. 1a).

* Corresponding author.
E-mail address: josemaria.minguez@ehu.es (J.M. Mínguez).

1350-6307/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engfailanal.2010.08.013

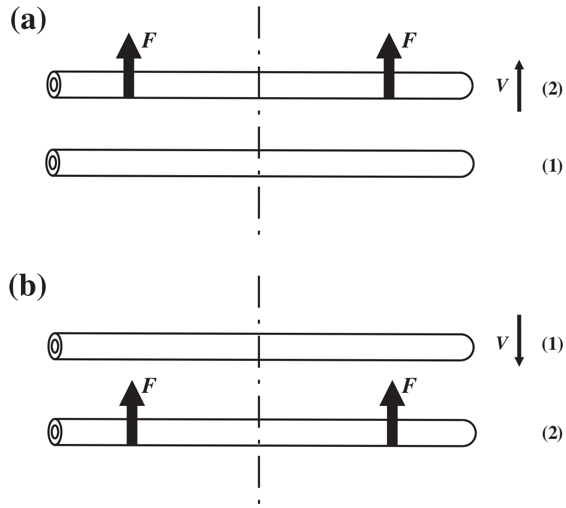

Fig. 1. (a) Stationary bar suddenly taken by forces F and communicated a speed V, (b) moving bar suddenly struck and stopped by forces F.

From the momentum balance

$$MV_1 = (M + m)V \tag{1}$$

together with the conservation of energy

$$\frac{1}{2}MV_1^2 = \frac{1}{2}(M + m)V^2 + U \tag{2}$$

the strain energy due to the loss of kinetic energy is

$$U = \frac{1}{2}mV^2 \tag{3}$$

Similarly, in the second case, assuming that the bar is struck at speed V by a much heavier object and that at a certain instant the bar is suddenly stopped by forces F (see Fig. 1b), the conservation of energy requires that the loss of kinetic energy be transformed into strain energy and so

$$U = \frac{1}{2}mV^2 \tag{4}$$

In the two cases considered the strain energy is represented by similar Eqs. (3) and (4), although in Eq. (3) V is the velocity at which the bar, initially stationary, suddenly starts moving, whereas in Eq. (4) V is the velocity from which the bar is suddenly stopped. In both cases the kinetic energy lost in the collision is transferred to all the colliding elements as strain energy. So the strain energy taken by the bar will be a fraction of U, which may be expressed as

$$U_b = \beta mV^2 \tag{5}$$

where $0 < \beta < 1/2$. This is going to be the energy employed in bending the bar.

3. Bending stress theory

In the two cases the bar is subjected to its own inertia force against the forces F and experiences bending according to the diagram described in Fig. 2. The inertia force (which is $-m \times$ acceleration) is a uniformly distributed load w acting along the length of the bar, since the bar mass is uniformly distributed. Then, as the two symmetrical supporting forces F equilibrate the total inertia force,

$$F = wL/2 \tag{6}$$

Now the relationship between the strain energy and the bending stresses depends on the position of the supporting reaction forces, which is given by the distance $A = \alpha L$ in Fig. 2.

The bending deflection mode and moment distribution along the bar depends very much upon where it is supported as shown in Fig. 2. The two extreme cases are shown, that is, when the forces F are simultaneously applied at both ends (i.e. when $\alpha = 0$) and when the forces are exerted at the central position (i.e. when $\alpha = L/2$) in Figs. 3 and 4.