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Nano-mechanics based modeling of lifetime distribution of quasibrittle structures

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ABSTRACT

The statistics of structural lifetime under constant load are related to the statistics of structural strength. The safety factors applied to structural strength must ensure failure probability no larger than 10^{-6}, which is beyond the means of direct verification by histogram testing. For perfectly brittle materials, extrapolation from the mean and variance to such a small tail probability is no problem because it is known that the Weibull distribution applies. Unfortunately, this is not possible for quasibrittle materials because the type of cumulative distribution function (cdf) has been shown to vary with structure size and shape. These are materials with inhomogeneities and fracture process zones (FPZ) that are not negligible compared to structural dimensions. A probabilistic theory of strength of quasibrittle structures failing at macro-crack initiation, which can be experimentally verified and calibrated indirectly, has recently been deduced from the rate of jumps of atomic lattice cracks governed by activation energy barriers. This paper extends this nano-mechanics based theory to the distribution of structural lifetime. Based on the cdf of strength and a power law for subcritical crack growth rate, the lifetime cdf of quasibrittle structures under constant loads is derived. The lifetime cdf is shown to depend strongly on the structure size as well as geometry. It is found that, for the creep rupture case, the mean structural lifetime exhibits a very strong size effect, much stronger than the size effect on the mean structure strength. The theory also implies temperature dependence of the lifetime cdf. For various quasibrittle materials, such as industrial ceramics and fiber composites, it is demonstrated that the proposed theory correctly predicts the experimentally observed deviations of lifetime histograms from the Weibull distribution.

1. Introduction

Many engineering structures, such as dams, nuclear structures and large bridges or buildings, must be designed against an extremely low failure probability under design load P during their service lifetime τ, i.e. $P_f(P,\tau) < 10^{-6}$ [17,30,33]. If such a low failure probability is required, it is impossible to determine the design lifetime by histogram testing. Therefore, it is imperative to develop a physically based probabilistic theory to predict the cumulative distribution function (cdf) of lifetime, so that it would be feasible to calibrate it experimentally.

The type of cdf of structural lifetime is well known for perfectly brittle structures, for which the failure is triggered by one negligibly small representative volume element (RVE) of material. In that case, the weakest-link model with an infinite number of links applies, and so the lifetime cdf must follow the Weibull distribution. This study focuses on structures consisting

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of quasibrittle materials, which include fiber composites, concretes, rocks, stiff soils, foams, sea ice, consolidated snow, bone, tough industrial and dental ceramics and many other materials on approach to nano-scale. These are with brittle constituents materials in which the inhomogeneities, and thus the RVE, are not negligible compared to the structure size. It has been demonstrated that the behavior of quasibrittle structures transits from quasi-plastic to brittle with increasing structure size [1,2]. Such a transition has serious consequences for structural reliability and lifetime prediction.

Tremendous efforts have been devoted to study the structural lifetime of many engineering materials in a deterministic framework. Tobolsky and Eyring [46] were first to adopt the concept of activation energy to make deterministic predictions of the mean lifetime of polymeric materials under constant loads. A similar deterministic model was developed by Zhurkov [50,51] to study the structural lifetime of polymers, alloys, and non-metallic crystals. This model neglects restorations of the ruptured interatomic bonds and thus gives unreasonably short lifetimes for failures at low stress [50], and for zero stress it even gives a finite lifetime. Hsiao et al. [22] employed a more general deterministic model which takes into account of the restorations of ruptured bonds. A comprehensive review of these deterministic models can be found in [21]. However, all these models derive the structural lifetime at the macro-scale directly from the frequency of rupture of interatomic bonds, and the multi-scale nano-macro transition is lacking. Furthermore, these deterministic models do not predict the type of lifetime cdf, which is essential for reliability-based design of engineering structures.

Meanwhile, significant advances have been made for various statistical models for structural lifetime of fibrous materials, unidirectional fiber composites and ceramics [15,16,45,37–39,31,29]. For unidirectional fiber composites, Coleman [15,16] first proposed a general lifetime distribution function to model the lifetime cdf of single fibers for a given loading history, which was based on the infinite weakest-link model. This model was subsequently adopted for the study of the lifetime statistics of fiber bundles and unidirectional fiber composites based on some chosen empirical rule of load redistribution among the fibers after breaks [37,38,45,23,29]. However, an infinite weakest-link model was used or implied, which was not physically justified because of non-negligible inhomogeneities, as evidenced by systematic deviations of measured histograms from the classical Weibull distribution [42,48,43]. Furthermore, simplified load sharing rules used for the fiber bundle model, such as the equal load sharing and the local load sharing [32,40], generally lack a physical basis and lead to questionable types of lifetime cdf. For ceramic materials, a more general approach has been adopted, in which the structural strength and lifetime are interrelated by the crack growth law [31,28]. In these models, the strength cdf was often assumed to follow the classical Weibull distribution, which inevitably leads to the Weibull distribution of structural lifetime. However, extensive experimental evidence shows both the strength and lifetime histograms of various ceramic materials to deviate from the Weibull distribution [27,34,31].

The objective of this paper is to present a nano-mechanics based theory for lifetime distribution of quasibrittle structures. A constant sustained load (as in the creep-rupture test) is considered, but extensions to other monotonic loading histories would be straightforward. Attention is limited to the broad class of structures that fail as soon as one macro-crack initiates. The theory will be validated by the optimum fits of lifetime histograms of various quasibrittle materials such as fiber composites and industrial ceramics.

2. Review of strength distribution of one RVE

For concrete, industrial and dental ceramics, fibers and fiber composites, it has been observed that the strength histograms consistently deviate from the two-parameter Weibull distribution [27,34,31,35]. It has recently been found that the problem lies in the tacit assumption that the weakest-link model which underlies the Weibull statistics of strength has infinitely many links [8,9]. The strength cdf of quasibrittle structures of positive geometry (i.e. structures that fail at crack initiation) should rather be modelled by a weakest-link model with a finite chain of finite-size RVEs. The reason is that the size of the RVE, which roughly coincides with the width of the fracture process zone (FPZ) at crack tip, is not negligible compared to the structure size D. This is the salient feature of quasibrittle structures.

A probabilistic theory for the cdf of strength of the broad class of quasibrittle structures failing at macro-crack initiation has recently been derived on the basis of breaks of interatomic bond pairs [8,9]. It was further refined based on atomistic fracture mechanics [5,6]. In the refined theory, a nano-crack is considered to propagate by random jumps through either a regular atomic lattice or through a disordered nano-structure. These jumps are governed by the activation energy barriers separating a series of numerous metastable states on the surface of the free energy potential of the nano-structure.

When the nano-crack advances by one atomic spacing in the atomic lattice or by one nano-inhomogeneity in a disordered nano-structure, the energy release increment must correspond to the change of activation energy barrier. Applying the equivalent LEFM (linear elastic fracture mechanics) to the nano-crack propagation, the energy release increment can be expressed as a function of the remote stress applied on the nano-structure [5,6].

Since the crack jumps by one atomic spacing or one nano-inhomogeneity are numerous and thus very small, the activation energy barrier for a forward jump differs very little from the activation energy barrier for a backward jump. Therefore, the jumps of the state of the nano-structure, characterized by its free energy potential, must be happening in both directions, albeit with different frequencies. After a certain number of jumps of the nano-crack tip, the length of the nano-crack reaches a critical value at which the crack loses its stability and propagates dynamically, causing a break of the nano-structure.

Since, at nano-scale, it may generally be assumed that each jump is independent (i.e., the frequency of the jump is independent of the particular history of breaking and restoration sequences that brought the nano-crack to the current length) [25], the failure probability of the nano-structure is proportional to the sum of the frequencies of all the jumps that cause its