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An extended (fractal) Overlapping Crack Model to describe crushing size-scale effects in compression

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ABSTRACT

The inherent microstructural disorder strongly influences the mechanical behaviour of heterogeneous materials such as concrete and rocks. Tensile and compression tests, in fact, evidenced a localization of strain and dissipated energy in the post-peak softening branch, with a consequent scale dependence of the stress–strain response. For this reason, the well-known *Cohesive Crack Model* and the recently proposed *Overlapping Crack Model* are useful tools for describing the size effects in tension and compression, respectively. In general, strain localization, damage and fracture, which are phenomena affecting the failure of concrete, are not rigorously interpretable in the framework of continuum mechanics. On the other hand, since the flaw and the aggregate distributions in quasibrittle materials are often self-similar (i.e. they look the same at different magnification levels), the microstructure may be correctly modelled by fractal sets. In this paper, the approach based on fractal geometry, that has profitably been applied for the tensile behaviour, is applied to obtain a fractal overlapping law from uniaxial compression tests. According to this approach, it is assumed that energy dissipation, stress and strain are not defined with respect to the canonical physical dimensions, though on fractal sets presenting noninteger physical dimensions. As a consequence, these classical parameters should be substituted by fractal quantities, which become the true material properties.

1. Introduction

The compression behaviour of concrete, and in particular the ultimate strength and the post-peak branch, have a predominant role in the design of concrete and concrete-based structures. Structural design, in fact, is usually conducted by comparing an action with a resistance evaluated on the basis of the ultimate strength. On the other hand, the post-peak behaviour is fundamental for a correct evaluation of the structural ductility, e.g., the ultimate axial deformation of columns or the rotational capacity of reinforced concrete beams. In this context, the size-scale effects play an important role, since the characteristic parameters of concrete are measured on specimens at the laboratory scale, that is far from the dimension of a real structure. The problem of the size-effect on the compression strength was deeply investigated in the literature [1,2], although it is very difficult to treat this phenomenon, since the strength and the behaviour of concrete highly depend on friction between concrete surfaces and loading platens, as well as on specimen slenderness [3,4].

The size-effects also influence the post-peak softening branch of the stress–strain diagram. Experimental tests evidenced that, in the softening regime, ductility (in terms of stress and strain) is a decreasing function of the size-scale and the slenderness, whereas it increases by increasing the friction [3,5]. The post-peak damage of concrete specimens subjected to uniaxial compression is characterized by the development of microcracking up to full fragmentation with a subsequent

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reduction of the applied load. Differently from the tensile behaviour, where the failure always takes place with the development of a tensile macrocrack independently of the size-scale and slenderness of the specimen, the mechanism leading to compression failure is not unique. By varying the size-scale and the slenderness, in fact, the actual failure mechanism of the specimens may vary from pure crushing to diagonal shear and splitting failures. Furthermore, the evaluation of the constitutive parameters is also complicated by some testing aspects, as, for example, the friction between concrete and loading platens. Based on these remarks, we can conclude that it is impossible to obtain a post-peak stress vs. strain constitutive law from uniaxial compression tests of concrete.

The stress–strain relationships usually assumed for calculation consider an energy dissipation within a volume and they are absolutely ineffective in the description of the size-scale effects. On the contrary, a close observation of the stress vs. post-peak deformation curves shows that a strong localization of deformations occurs in the softening regime, independently of the loading system, confirming the earlier results by van Mier [6]. The phenomenon of strain localization in compression, evidenced in many other experimental programmes on concrete and rocks [7–11], suggests that, in the softening regime, energy dissipation takes place over an internal surface rather than within a volume, in close analogy with the behaviour in tension. These two interconnected phenomena may explain the size-scale effects on structural ductility. Based on these remarks, Carpinteri et al. [12,13] recently proposed the *Overlapping Crack Model*, in order to model the process of concrete crushing using an approach analogous to the *Cohesive Crack Model* [14,15], which is routinely adopted for modelling the tensile behaviour of concrete. In tension, the localized displacement is represented by a crack opening, while in compression it would be represented by a material interpenetration. This new approach, also based on Fracture Mechanics concepts, assumes a stress–displacement law as a material characteristic for the post-peak behaviour of concrete in compression.

It is worth noting that the assumption of an energy dissipation over a surface is an effective idealization of a more complex mechanism characterized by diffused macrocracks after the coalescence of initial microcracks. This means that, globally, the energy dissipation is a surface-dominated phenomenon. An accurate description of such a reality cannot be done on the basis of the classical continuum mechanics. In the case of materials with heterogeneous microstructures, as, for example, concrete and rocks, singularities and inhomogeneities play in fact a fundamental role in the definition of the physical properties. For this reason, the classical mechanical parameters relying on regularity properties of the medium, as the nominal stress, the energy dissipation per unit volume and the nominal strain, become meaningless. In this context, Carpinteri [16,17], Carpinteri and Cornetti [18] and Carpinteri et al. [19,20] proved that fractal geometry [21,22] is a useful tool for giving a unified explanation to the scale-effects affecting the characteristic parameters of the *Cohesive Crack Model*. According to this approach, true scale-invariant material parameters have been obtained by abandoning the classical physical dimensions to advantage of noninteger ones. In the case of quasibrittle materials, the application of fractal concepts is motivated by the inherent self-similarity of the aggregate size distribution (see [20,23]). In addition, the hypothesis of damage domain showing noninteger physical dimensions received different experimental confirmations (see, for instance, Carpinteri et al. [24,25]). Applications of fractal concepts to the compression behaviour of concrete structures have been also proposed to give theoretical explanations to the size-scale effects on the compression strength [2], as well as on the dissipated energy density [26–28].

The present paper represents an attempt to obtain a material constitutive law for concrete in compression independent of the size-scale and slenderness of the specimen, by adopting approaches that are unusual for solid mechanics. Firstly, the effective applications of the fractal concepts to the tensile behaviour, proposed by Carpinteri et al. [19,20] for obtaining a scale-independent cohesive law, are reported. Then, the *Overlapping Crack Model*, which is an important step towards a complete description of the size-scale effects in compression, is presented. Finally, an overlapping fractal law is obtained for concrete specimens subjected to compression.

2. Scale-independent (fractal) Cohesive Crack Model

In this section, the results of the fractal analysis of damage domains performed by Carpinteri [16,17], Carpinteri and Cornetti [18] and Carpinteri et al. [19,20] in order to obtain a fractal cohesive model independent of the structural size, are summarized. It is well-known that strain and damage localization deeply influences the behaviour of concrete in tension. The development of a tensile macrocrack in the post-peak softening range, makes the use of Fracture Mechanics necessary in order to correctly model the actual failure mechanism. In particular, due to the nonlinear behaviour in the process zone, the *Cohesive Crack Model* introduced by Hillerborg et al. [14,15] is the most suitable model describing, in a satisfactory way, the damage localization in concrete and concrete-like structures [29,30]. This model is able to explain size effects in the case high stress gradients are present, i.e. in the case of tests on pre-notched specimens and specimens subjected to bending test [31–34]. On the other hand, relevant scale effects are encountered also in uniaxial tension tests (see [35–37]), where much smaller stress gradients are present. In this case, size effects should be inherent to the material behaviour rather than to the stress intensification. In particular, the tensile strength exhibits a reduction by increasing the structural size, whereas the fracture energy and the critical displacement are increasing functions of the structural size. A unified explanation of the scale-effects affecting these physical quantities is based on the fractal geometry approach by Carpinteri et al. [19,20]. In the case of materials with heterogeneous microstructure (e.g., concrete and rocks), singularities and inhomogeneities play a fundamental role in the definition of the physical properties. In these cases, constant values of the mechanical parameters can be obtained only if the classical physical dimensions are abandoned to advantage of noninteger physical