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Effects of second-order slip on the flow of a fractional Maxwell MHD fluid

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Abstract The magnetohydrodynamic (MHD) flow of a generalized Maxwell fluid induced by a moving plate has been investigated, where the second-order slip between the wall and the fluid in the wall is considered. The fractional calculus approach is used to establish the constitutive relationship model of the non-Newtonian fluid model. Exact analytical solutions for the velocity field and shear stress in terms of Fox H-function are obtained by means of the Laplace transform. The solutions for the generalized Maxwell second-order slip model without magnetic field, the MHD flow of generalized Maxwell flow without slip effects or first-order slip model can be derived as the special cases. Furthermore, the influence of the order of fractional derivative, the magnetic body force, the slip coefficients and power index on the velocity and shear stress are analyzed and discussed in detail. The results show that the velocity corresponding to flows with slip condition is lower than that for flow with non-slip conditions, and the velocity with second-slip condition is lower than that with first-order slip condition.

1. Introduction

There are many physical phenomenons actually with incomplete viscoelastic fluid in engineering and industry, such as polymer solutions, exotic lubricants, colloidal and suspension solutions, food stuffs, synthetic propellants, molten plastics and many others. These fluids have been modeled in a number of diverse manners with their constitutive equations varying greatly in complexity, among which the viscoelastic Maxwell fluid model has been studied widely (Fetecau and Fetecau, 2004; Tan and Masuoka, 2007; Jamil et al., 2011; Abbasbandy et al., 2014). The Maxwell model can be represented by a purely viscous damper and a purely elastic spring connected in series (Christensen, 1971), which has been proposed to describe the behavior of viscoelastic fluids, and has had some success in describing polymeric liquids, it being more amenable to analysis and more importantly experimental.

Rheological constitutive equations with fractional derivatives (Podlubny, 1999; Song and Jiang, 1998; Imran et al., 2017) have been proved to be a valuable tool to describe the behaviors of viscoelastic properties. The fractional derivative models of the viscoelastic fluids are derived from classical equations, which are modified by replacing the time derivative of an integer order by precisely non-integer order integrals or

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derivatives.With the development of research, the fractional derivative models of the viscoelastic fluids are concerned by considerable researchers. Song and Jiang, 1998 used the fractional calculus to analyze the experiment data of viscoelastic gum and obtained satisfactory results. Fetecau et al., 2008 investigated the unsteady flow of a second-grade fluid induced by the time-dependent motion of a plate between two side walls perpendicular to the plate. Xue et al., 2008 and Xue and Nie, 2009 discussed the exact solutions of the Rayleigh-Stokes problem for a heated generalized viscoelastic fluid in a porous half-space. Jamil et al., 2011 researched the unsteady flow of an incompressible Maxwell fluid with fractional derivative induced by a sudden moved plate, and analyzed the influence of the material and the fractional parameters on the fluid motion. Qi and Guo, 2014 studied a new heat conduction equation which is based on the time-nonlocal generalized of Fourier law, the exact solution of an initial-boundary value problem was studied and presented under series forms. Fan et al., 2015 presented an inverse problem to estimate parameters in generalized fractional Zener model based on the Bayesian method, and performed some examples to certify the validity of the method. Imran et al., 2017 investigated some natural convection flows of differential type fluids with Caputo fractional derivatives, and solved the velocity fields using Laplace transform method.

The assumption that a liquid adheres to a solid boundary so called no-slip boundary condition has been proved to be inadequate in several situations such as: the mechanics of thin fluids, flows in micro-channels, or flows of polymeric liquids with high molecular weight. For describing the slip that occurs at solid boundaries, a large number of models have been proposed. In recent years, Ebaid, 2008 studied the effects of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel. Jamil and Khan, 2011 considered the first order slip effect on fractional Maxwell fluids, and obtained the solutions of velocity field and shear stress. Vieru and Rauf, 2011 and Vieru and Zafar, 2013 investigated some Couette flows and Stokes flows of a Maxwell fluid with slip condition. The exact solutions of generalized Oldroyd-B fluid flow with the slip effects were obtained by Zheng et al., 2012. Han et al., 2015 presented a slip flow of a generalized Burgers' fluid between two side walls generalized by an exponential accelerating plate and a constant pressure, the analytical solutions are established and analyzed. Akbar and Khan, 2016a given the numerical study of carbon nanotubes suspended magnetohydrodynamic(MHD) stagnation point flow over a stretching sheet with convective slip. Shakeel et al., 2016 studied the flows of an Oldroyd-B fluid under the consideration of slip condition at the boundary, the fluid motion is generated by the flat plate which has a translational motion in its plane with a time-dependent velocity. Hayat et al., 2016 investigated the unsteady MHD flow over exponentially stretching sheet with velocity and thermal slip boundary conditions, and analyzed various pertinent parameters on the axial velocity and temperature distributions.

The motivation of the present study is to find the velocity field and shear stress corresponding to the second-order slip flow of a generalized Maxwell fluid over a plate with the assumption of low-magnetic Reynolds number. The fractional calculus approach is used to establish the constitutive relationship of the non-Newtonian fluid model. The exact solutions, obtained by means of a finite Fourier sine transform and a discrete Laplace transform, are presented using series forms in terms of the Fox H-function. The solutions of generalized Maxwell fluid without magnetic field effect, or the first-order slip, or without slip can be recovered from our solutions. Finally, the influence of the material, slip, fractional, magnetic and index parameters on the motion of generalized Maxwell fluids are underlined by graphical illustrations. The difference among generalized Maxwell fluid and classical Maxwell fluid models is also analyzed.

2. Governing equations

The constitutive equations for an incompressible fluid are given by

$$\nabla \cdot \mathbf{V} = 0, \quad \rho \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \nabla \cdot \mathbf{T} + \rho \mathbf{b}, \tag{1}$$

where $\mathbf{T}$ is the Cauchy stress tensor, $\mathbf{V}$ is the velocity vector, ρ is the constant density of the fluid, $\mathbf{b}$ is the body force field. There we consider the MHD fluid, which is affected by magnetic field $\mathbf{B}_0$. We neglected the induced magnetic field by assuming very large magnetic diffusivity. It is also assumed that no electric field is applied.

The constitutive equation of an incompressible Maxwell fluid is written in the form (Fetecau and Fetecau, 2004; Tan and Masuoka, 2007; Han et al., 2015):

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \quad \mathbf{S} + \lambda \frac{\mathrm{D}\mathbf{S}}{\mathrm{D}t} = \mu \mathbf{A}, \tag{2}$$

where

$$\frac{\mathrm{D}\mathbf{S}}{\mathrm{D}t} = \frac{\mathrm{d}\mathbf{S}}{\mathrm{d}t} + \mathbf{V} \cdot \nabla \mathbf{S} - \mathbf{L}\mathbf{S} - \mathbf{S}\mathbf{L}^{\mathrm{T}}, \tag{3}$$

and $-p\mathbf{I}$ denotes the indeterminate spherical stress, $\mathbf{S}$ is the extra-stress tensor, $\mathbf{A} = \mathbf{L} + \mathbf{L}^{\mathrm{T}}$ is the first Rivlin–Ericksen tensor, $\mathbf{L}$ is the velocity gradient, μ, λ are material constants, known as the viscosity coefficient, the relaxation times, respectively. If $\lambda = 0$, the Eq. (2) is the constitutive equation of a Newtonian fluid.

We consider the fluid is permeated by an imposed uniform magnetic field $\mathbf{B}_0$ which acts in the positive y-coordinate (Jamil et al., 2013; Akbar and Khan, 2016b; Akbar et al., 2016a, Akbar et al., 2016b,Akbar et al., 2016c), by assuming a very small magnetic Reynolds number, the induced magnetic field is neglected. Hence, the Lorentz force caused by the external magnetic field can be represented as $(-\sigma B_0^2 u, 0, 0)$. B_0 is the magnitude of $\mathbf{B}_0$ and σ is the electrical conductivity of the fluid, u denotes the x-component of the fluid velocity. The velocity and shear stress for one dimensional flow take the form:

$$\mathbf{V} = u(y,t)\mathbf{i}, \quad \mathbf{S} = \mathbf{S}(y,t), \tag{4}$$

where $\mathbf{i}$ is the unit vectors in the x-direction. For this flow, the constant of incompressible is automatically satisfied. Taking account of the initial condition $\mathbf{S}(y,0) = 0$ and in the absence of pressure gradient in the x-direction, the motion equation of the generalized Maxwell fluid is:

$$(1 + \lambda \partial_t)\tau(y,t) = \mu \partial_y u(y,t), \tag{5}$$

$$(1 + \lambda \partial_t)\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - M(1 + \lambda \partial_t)u, \tag{6}$$