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Finite-time synchronization of inertial neural networks

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Abstract In this paper, the finite-time synchronization of inertial neural networks is investigated. First, to realize synchronization of the master–slave system, continuous and discontinuous controllers are designed, respectively. By constructing Lyapunov function and using inequalities, some effective criteria are provided to realize synchronization in finite time. Furthermore, in order to achieve synchronization with a fast speed, a new switching controller is presented, and the upper bounds of the settling time of synchronization are estimated. Finally, several numerical simulations are presented to demonstrate the validity of the theoretical results and the effectiveness of the proposed method.

1. Introduction

In the past decades, the neural network has arisen many researchers' attention, owing to the fact that it can be widely applied in signal and image processing, financial industry, pattern recognition, control and optimization problems (see Kwok and Smith, 2000; Zhou and Cao, 2004; Wang, 1993). Such applications severely depend on the dynamical behaviors of the neural network, therefore, the investigation of dynamical behaviors is an important step for practical design of neural networks. Up to now, many results on stability of equilibrium points and synchronization analysis of the neural networks have been published, we referee the interested reader to the papers of Cao and Zhou (1998), Wen et al. (2013), Li and Liao (2011), Fang (2015), He and Cao (2009) and their references.

It is worth noting that most of the previous works mainly focused on neural networks with only first order of the state, whereas it is great importance to introduce an inertial term into the neural networks. The inertial network with second order of the state, and there are some literatures that have studied the second order equation, such as Alaba and Ogundare (2015), Tunc (2011), and Tunc and Tunc (2015). Comparing to electronic neural networks with the standard resistor–capacitor variety, Babcock and Westervelt (1986) showed that the dynamics could be complex when the neuron couplings included an inertial nature. In fact, there exist evident biological backgrounds for introducing an inertial term into the standard neural networks system (see Angelaki and Correia, 1991; Ashmore and Attwell, 2014). At present, many conclusions of inertial neural networks have been obtained. For example, by employing matrix measure strategies, Tu et al. (2016), Cao and Wan (2014) studied the dissipativity, stability and synchronization of inertial delayed neural networks, respectively. The globally exponentially stable in Lyapunov sense of inertial neural networks was considered, by using

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analysis method and inequality technique in Ke and Miao (2013), Zhang and Quan (2015), and Qi et al. (2015). Meanwhile, Hu and Cao (2015) investigated the pinning synchronization of coupled inertial delayed neural networks, by utilizing matrix measure strategy and Lyapunov function approach. Moreover, in Zhang and Li (2015), the exponential stability of inertial BAM neural networks with time-varying delay was arrived via periodically intermittent control.

The synchronization of the master–slave system has been extensively investigated due to its applications in communication security. Synchronization indicates that the states of the slave system converge those of the master system. Nowadays, based on different convergence time, synchronization has been classified into two types: one is the infinite time synchronization, such as asymptotic or exponential synchronization, the other is finite-time synchronization. In contrast to the commonly concept of asymptotic synchronization, the finite-time synchronization requires the master system and the slave system remain completely identical after some finite time, which is called the settling time. In review of past research, finite-time synchronization for different types of neural networks have been widely studied. Finite-time synchronization of complex dynamical networks is considered by Chen and Lü (2009), Mei et al. (2013), and Hu et al. (2014) presented the finite-time synchronization of delayed neural networks, based on delayed feedback control. Furthermore, in Liu et al. (2015), via nonsmooth analysis, finite-time synchronization of complex networks is realized. Bao and Cao (2016) considered the finite-time generalized synchronization of nonidentical delayed chaotic systems, and finite-time synchronization of fractional-order memristor-based neural networks with delays is researched, by using Laplace transform and the generalized Gronwall's inequality in Velmurugan et al. (2016).

Motivated by the above analysis, this paper investigates the finite-time synchronization of inertial neural networks, the problem is still open, and is no report in published literatures. Therefore, we will try to solve this challenging and important problem. The main contribution of this paper lies in following aspects. (1) Finite-time synchronization of the inertial neural networks is first introduced. By utilizing continuous $(0 < \eta < 1)$ and discontinuous $(\eta = 0)$ controllers, several new criteria are obtained to guarantee the synchronization of master–slave system. (2) A new switching controller that includes continuous and discontinuous is designed to receive the fast convergence speed. (3) The upper bounds of the settling time of synchronization are estimated.

The rest of this paper is organized as follows. Problems description and preliminaries are given in Section 2. In Section 3, several criteria for finite-time synchronization of inertial neural networks are obtained by designing different types of controllers. In Section 4, several numerical simulations are presented to show the effectiveness of the theoretical results. A short conclusion is given in Section 5.

Notations. In this paper, $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times n}$ is the set of all $n \times n$ real matrices, $C > 0$ means that the matrix C is symmetric and positive definite, C^T denotes the transpose of matrix C, $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalues of a real symmetric matrix, respectively. We denote the vector norm: $\|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{x^T x}$, matrix norms: $\|A\|_2 = [\lambda_{\max}(A^T A)]^{1/2}$, sign$(\cdot)$ is the sign function, diag$(\cdot)$ stands for diagonal matrix, I_n denotes the identity matrix.

2. Problem description and preliminaries

In this paper, we consider the following inertial neural networks:

$$\frac{d^2 x_i(t)}{dt^2} = -a_i \frac{dx_i(t)}{dt} - b_i x_i(t) + \sum_{j=1}^{n} c_{ij} f_j(x_j(t)) + I_i, \tag{1}$$

where $i = 1, 2, \ldots, n$, the second derivative of $x_i(t)$ is called an inertial term of system (1), $x_i(t)$ denotes the state variable of the ith neuron at time t, a_i and b_i are positive constants, b_i denotes the rate at which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external input, c_{ij} is constant and denotes the connection strength of the jth neuron on the ith neuron, f_j denotes the activation function of jth neuron at time t, I_i is an external input for ith neuron.

To establish our main results, it is necessary to give the following assumption for the system (1).

(H) Suppose that the activation function $f(\cdot)$ satisfies the following condition,

$$\|f(x) - f(y)\| \leqslant \|F(x-y)\|, \qquad \forall x, y \in \mathbb{R}^n,$$

where $F \in \mathbb{R}^{n\times n}$ is a known constant matrix.

Let the following variable transformation be:

$$y_i(t) = \frac{dx_i(t)}{dt} + \xi_i x_i(t), \qquad \xi_i \in \mathbb{R}, \qquad i = 1, 2, \ldots, n,$$

then system (1) can be written as:

$$\begin{cases} \frac{dx_i(t)}{dt} = & -\xi_i x_i(t) + y_i(t), \\ \frac{dy_i(t)}{dt} = & -\alpha_i y_i(t) - \beta_i x_i(t) + \sum_{j=1}^{n} c_{ij} f_j(x_j(t)) + I_i, \end{cases} \tag{2}$$

where $\alpha_i = a_i - \xi_i, \beta_i = b_i + \xi_i(\xi_i - a_i)$.

Denote $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$, $\Xi = \mathrm{diag}\{\xi_1, \xi_2, \ldots, \xi_n\}$, $A = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, $B = \mathrm{diag}\{\beta_1, \beta_2, \ldots, \beta_n\}$, $C = (c_{ij})_{n\times n}$, $I = (I_1, I_2, \ldots, I_n)^T$. Hence the system (2) can be written as matrix–vector form:

$$\begin{cases} \frac{dx(t)}{dt} = & -\Xi x(t) + y(t), \\ \frac{dy(t)}{dt} = & -Ay(t) - Bx(t) + Cf(x(t)) + I. \end{cases} \tag{3}$$

In this paper, we will make master–slave chaotic neural networks achieve synchronization in finite time by designing some effective controllers. For simplicity, we refer to model (3) as the master system, the slave system is described as follows:

$$\begin{cases} \frac{du(t)}{dt} = -\Xi u(t) + v(t) + U_1(t), \\ \frac{dv(t)}{dt} = -Av(t) - Bu(t) + Cf(u(t)) + I + U_2(t), \end{cases} \tag{4}$$

where $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$ and $v(t) = (v_1(t), v_2(t), \ldots, v_n(t))^T$ are states of the controlled system. $U_1(t), U_2(t)$ are the appropriate control inputs that will be designed.

Let the errors be $e(t) = u(t) - x(t), \hat{e}(t) = v(t) - y(t)$, we can derive the following error system

$$\begin{cases} \frac{de(t)}{dt} = & -\Xi e(t) + \hat{e}(t) + U_1(t), \\ \frac{d\hat{e}(t)}{dt} = & -A\hat{e}(t) - Be(t) + C(f(u(t)) - f(x(t))) + U_2(t). \end{cases} \tag{5}$$

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