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ORIGINAL ARTICLE

Optimal solution of integro-differential equation of Suspension Bridge Model using Genetic Algorithm and Nelder-Mead method

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Abstract In this paper, a fourth order integro-differential arising in the modeling of suspension bridge. A hybrid Genetic Algorithm (GA) and Nelder-Mead (NM) optimization technique is employed along with collocation finite element method and Optimal Homotopic Asymptotic Method. The solutions are compared with existing solutions. A excellent comparison is obtain results are shown, graphs and tables are shown.

1. Introduction

Evolutionary computation (EC) is one of the most widely used optimization technique around the globe. These algorithms are inspired by Darwin's theory of evolution i.e. survival of the fittest. Out of many EC techniques Genetic Algorithms (GA) takes a prestigious position. GA is an artificial intelligent system which uses some initial guess called chromosomes to eventually evolutions a set of solutions in the neighborhood of Global optimal value. GA doesn't always give a very accurate solutions but its advantage is it always tends to converge to global optimum value. To enhance the accuracy of this method Mastorakis (2005) presents a new hybrid optimization technique for nonlinear equations. Nelder-Mead method is now widely used in non-linear optimization problems (Yildiz et al., 2003, 2016; Nawaz et al., 2015, 2016; Sarakhsi et al., 2016; Yildiz, 2013; Zeeshan et al., 2016; Gökdağ and Yildiz, 2012). Nelder-Mead Algorithm are downhill simplex method which are very accurate scheme, but it may converge to local optimal value depending upon choice of initial guess. This hybrid scheme tends to over come such hurdles. Scientists applies hybrid schemes to good effects over the years Hybrid particle swarn optimization technique was employed by Yildiz (2012) for the structural design in cars. A hybrid differential evolution algorithm is worked out by Yildiz (2013). Yildiz and Solanki (2012) uses multi object optimization of vechicle crashworthing. They uses particle swarn optimization for the process. Yildiz (2012) also studied population base optimization algorithm and Hybrid differential evolution algorithm (2013) for variety of problems in automobile industry. A few other hydrid algorithms are used to good effect are discussed in (Yildiz et al., 2004; Yildiz, 2013; Yildiz et al., 2016;

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Yildiz, 2013; Yildiz and Saitou, 2011; Yildiz, 2013) and therein. The solution obtained from Genetic Algorithms serves as initial guess for Nelder-Mead method which tends to converge to optimal value which is closest to it (Global) optimum.

Analytic solutions of non-linear problems are challenging affair, specially for the likes of integro-differential equations. Nafay introduced perturbation methods, opening a new gate way toward the analytic solution of linear and non-linear problems. These methods grow rapidly and have an ever growing literature. But, these methods have some limitations like they may need some small or large parameter and especially in strongly nonlinear systems these classical methods fails. Therefore, to overcome such issues new powerful analytical tools were developed, such as HPM, VIM (Siddiqi and Iftikhar, 2015), Homotopy Analysis Method (HAM) (Liao, 2003) are introduced. These effective analytical tools can deal with strong non-linearity and no small or large parameters are involved. In the recent past many scientist checked HAM's effectiveness and efficiency successfully (Hayat et al., 2016; Freidoonimehr et al., 2016; Ellahi et al., 2015, 2016; Rashidi et al., 2015; Bhatti et al., 2017; Ellahi et al., 2015; Shehzad et al., 2016; Nawaz et al., 2015; Zeeshan et al., 2017; Bhatti et al., 2016). To improve the accuracy Marinca and Herisanu (2008) developed techniques which uses optimization by hybridizing them with advanced perturbation techniques. Which not even improves its accuracy but also make the methods fast. Zeeshan et al. (2014) combine optimal HAM with Hybrid GA and NM method and obtain efficient solution of flow problem through coaxial cylinders.

The numerical behavior of the integro-differential equations is a challenging problem. In the present paper, Genetic Algorithm and Nelder-Mead are hybridized with finite element method and optimal homotopy analysis method to solve the integro-differential equation. The numerical experiment shows the method works efficiently. In Section 2 the mathematical formulation for the problem of multiple phase suspension bridge is shown. In Section 3, OHAM and Optimization techniques used are briefly discussed. In Section 4, the two problems are solved analytically and numerically by taking constant and variable live loads. Solution is described graphical and in tabular form. Comparison is established with existing solutions.

2. Mathematical formulation

Pugsley (1968) develop a model for three span suspension bridge given as

$$EIv_i^{(iv)} - (T_p + T_g)v_i'' = p(x) - \frac{W}{T_g}T_p, \tag{1}$$

where

$$T_p = \frac{E_c A_c}{L_c}\left(\frac{W}{T_g}\right)\sum_{i=1}^{3}\int_{l_{l_i}} v_i(x)dx, \tag{2}$$

w is the dead load weight of the bridge, E is the modulus of elasticity, T_g is tension of horizontal cable and taken as constant throughout the bridge, E_c is modulus of elasticity, A_c is cross sectional area of the cable and L_c is the length of spans, given by

$$L_c = \int_0^l \left(\frac{ds}{dx}\right)dx, \tag{3}$$

where, $l = l_1 + l_2 + l_3$ the total length of all spans.

Assuming the live load is relatively small compared to the dead load than $T_p \ll T_g$, and hence the Eq. (1) becomes linear fourth order integro-differential equation,

$$EIv_i^{(iv)} - T_g v_i'' = p(x) - \frac{W}{T_g}T_p, \tag{4}$$

along with boundary conditions

$$v_i(x) = v_i''(x) = 0, \tag{5}$$

at the ends of each span.

3. Solution methodology

In present study solutions are obtained numerically using Finite Element Method (FEM) and analytical using Optimal Homotopic Asymptotic Method (OHAM). The solutions are then optimized using Hybrid GA and NM scheme.

3.1. Optimal Homotopic Asymptotic Method (OHAM)

We consider a general differential equation as

$$L(u) + N(x, u, u', u'', \ldots) + f(x) = 0, \quad B(u, u', \ldots) = 0 \tag{6}$$

where $L(u)$ is linear operator, $N(x, u, u', u'', \ldots)$ is the equation excluding linear operator, $B(u, u', \ldots)$ are boundary or initial conditions and $f(x)$ is the forcing function. Applying the homotopy on the equation we have,

$$(1-p)(L(u) + f(x)) + H(K(k), p)(L(u) + N(x, u, u', u'', \ldots) + f(x)), \tag{7}$$

where u_0 is initial guess which satisfies boundary conditions and p is embedding parameter varies from 0 to 1, such that for $p = 0, L(u) + f(x) = 0$ and deforms to $L(u) + N(x, u, u', u'', \ldots) + f(x) = 0$ at $p = 1$. Hence, $H(K, p)$ is defined such that at $p = 0, H(K(k), p) = 0, p = 1, H(K(k), p) = 1$. $K(k)$ are function of k which is the optimization parameter.

Taking

$$u = u_0 + pu_1 + p^2u_2 + \cdots. \tag{8}$$

For different powers of p we have system of linear equation with embedding optimization parameters. Solution is obtained by putting $p = 1$, forming

$$u = u_0 + u_1 + u_2 + \cdots. \tag{9}$$

Optimization parameter k are then optimized for minimal residue value using GA and NM technique.

3.2. Optimization

Genetic Algorithms GAs are inspired by Darwin's theory of evolution. There are five main discussion points in the whole procedure, **Fitness Functions** are the functions which are meant to be minimize, they are of particular type which satisfies a large search space. **Population**: The size of initial population i.e. how many set of individuals are there in every generation called chromosomes. Also, type of population choice is important for example, if constraints are numerical values