ARTICLE IN PRESS

Journal of the Association of Arab Universities for Basic and Applied Sciences (2016) **xxx**, xxx–xxx

University of Bahrain

Journal of the Association of Arab Universities for Basic and Applied Sciences

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www.sciencedirect.com

ORIGINAL ARTICLE

Fully developed flow of non-Newtonian fluids in a straight uniform square duct through porous medium

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Received 13 August 2014; revised 7 February 2016; accepted 3 April 2016

KEYWORDS

Couple stress fluid;
Jeffrey fluid;
Square duct;
Porous medium;
Finite difference method

Abstract In this paper, we have studied the flow of incompressible fluids in a straight square duct through the porous medium. The couple stress fluid model and Jeffrey fluid model are considered separately to study the flow properties. The governing partial differential equations have been solved numerically using finite difference method in each case. In both the cases, the variation of different flow parameters on the fluid velocity is illustrated graphically and the numerical results for the volume flow rate have been presented through tables. It is observed that, the velocity and volume flow rate decrease with an increase in couple stress parameter and porosity parameter, while the velocity and volume flow rate increase with an increase in Jeffrey parameter and pressure gradient.

1. Introduction

The study of non-Newtonian fluids is very important because of its applications in several industrial and engineering processes. Many materials such as drilling mud, blood, ketchup, tooth-paste, certain oils and greases, polymer melts and many other emulsions have been treated as non-Newtonian fluids. Due to vast variety in the physical structure of real fluids, it is not easy to propose a single constitutive equation which exhibits all properties of real fluids. Therefore, a number of non-Newtonian fluid models have been proposed to predict the behavior of real fluids. Due to its diverse applications, many authors have studied the non-Newtonian fluid flows in different geometries (see Radhakrishnamacharya, 1977; Rao, 1999; Vajravelu et al., 2002; Fetecau and Fetecau, 2005; Kothandapani and Srinivas, 2008; Firouzi and Hashemabadi, 2009; Khan et al., 2010; Liu et al., 2011; Mukhopadhyay and Bhattacharyya, 2012; Ellahi, 2013; Devakar, 2013; Hayat et al., 2013a,b). A comprehensive review of fluids of differential type and their applications is made by Ellahi (2014).

The couple stress fluid model initiated by Stokes (1984) presents a simple generalization of the classical viscous Newtonian model which allows for polar effects such as the presence of couple stresses and body couples in the fluid medium. The important feature of this fluid is that, the stress tensor is not symmetric. The equations governing the couple stress fluid flow are of higher order than the classical Navier–Stokes equations and offer challenges to the researchers working in this field. The study of couple stress fluid is very useful in understanding

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Peer review under responsibility of University of Bahrain.

http://dx.doi.org/10.1016/j.jaubas.2016.04.001
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various physical problems because it possesses the mechanism to describe the rheological complex fluids such as liquid crystals, lubricants containing small amount of polymer additive and human blood. In view of this, several researchers have made their contributions to the study of couple stress fluid flow problems. Devakar and Iyengar (2008) have discussed Stokes' first and second problems for an incompressible couple stress fluid. Srinivasacharya et al. (2009) have discussed the flow and heat transfer of couple stress fluid in a porous channel with an expanding and contracting wall. Ramana Murthy et al. (2010) have presented finite difference solution for MHD flow of couple stress fluid between two concentric rotating cylinders with porous lining. Devakar and Iyengar (2010) have studied the run up flow of an incompressible couple stress fluid between parallel plates. Farooq et al. (2013) have studied the non-isothermal Poiseuille flow between two heated parallel inclined plates using incompressible couple stress fluid. Devakar et al. (2014) have obtained analytical solutions of couple stress fluid flows between parallel plates with slip boundary conditions. Recently, Ramesh and Devakar (2015) have discussed the effects of magnetic field and heat transfer on the peristaltic flow of an incompressible couple stress fluid through porous medium in an inclined asymmetric channel.

Another non-Newtonian fluid model that has attracted the attention of researchers in fluid dynamics is the Jeffrey fluid model which describes the effects of the ratio of relaxation to retardation times and retardation time. Kothandapani and Srinivas (2008) have studied the peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Qayyum et al. (2012) have discussed the unsteady squeezing flow of Jeffery fluid between two parallel disks. Akbar and Nadeem (2012) have analyzed the simulation of variable viscosity and Jeffrey fluid model for blood flow through a tapered artery with a stenosis. Ellahi et al. (2013) have discussed three-dimensional stretched flow of Jeffrey fluid with variable thermal conductivity and thermal radiation.

The flow through porous medium is of fundamental importance in geomechanics, biomechanics and industry. The applications in which flow through a porous medium is mostly prominent are filtration of fluids, seepage of water in river beds, movement of underground water and oils, functioning of human lung, physiological fluid flow in bile duct and gallbladder with stones, and flow of blood through small blood vessels. Aforementioned applications inspired the researchers to investigate the flows through porous medium in different geometries. Afifi and Gad (2001) have made a theoretical study on the interaction of peristaltic flow with pulsatile magnetofluid through a porous medium. Murthy et al. (2004) have discussed the effect of double stratification on free convection in Darcian porous medium. Zeeshan and Ellahi (2013) have studied the effect of heat transfer and magnetic field on the third grade fluid in a pipe with porous space. A few more studies on the flows through porous medium for diverse situations are made by Prasad and Kumar (2011), Tripathi (2002), Ellahi et al. (2013) and Sheikholeslami et al. (2014).

The flow of fluid in a square duct is one of the most important flows in fluid mechanics because of its applications in industry and medicine. The applications include supply of fluids via pipe lines in the oil and petrochemical industries, food production, the fabrication of chemical materials, medical applications, and the injection of polymeric materials. The flow through a straight duct of square cross section was reported by Williams and Baker in 1966 (Johnson, 1998). Rahman and Ahmad (1982) have presented finite element analysis of axial flow with heat transfer in a square duct. Cook and Rahman (1986) have presented exact solutions of the temperature and velocity distributions for the Newtonian fluid flow through a square duct. Subsequently, many authors have studied the flow problems through the ducts of square cross section. Sayed-Ahmed (2000) discussed the laminar heat transfer for thermally developing flow of a Herschel–Bulkley fluid in a square duct. Beale (2005) studied the effect of mass transfer on Newtonian fluid in square duct. Adachi (2006) discussed the stability of natural convection in an inclined square duct with perfectly conducting side walls. Zhang et al. (2007) have presented the numerical study of flow of Oldroyd-3-Constant fluid in a straight duct with square cross-section. Lee (2008) studied the convective heat transfer to water near the critical region in a horizontal square duct. Norouzi et al. (2010) have investigated the inertial and creeping flow of a second-order fluid in a curved duct with a square cross-section. Heris et al. (2011) have made an experimental study on the forced convective heat transfer through square cross-sectional duct under laminar flow regime using CuO/water nanofluid. Tympel et al. (2012) have investigated the distortion of liquid metal flow in a square duct due to the influence of a magnetic point dipole. Sarma et al. (2014) have presented a numerical study for steady MHD flow of liquid metal through a square duct under the action of strong transverse magnetic field. Kun et al. (2014) have investigated experimentally the study of pseudoplastic fluid flows in a square duct of strong curvature. Ting and Hou (2015) have numerically investigated the convective heat transfer of water-based Al_2O_3 nanofluid flowing through a square cross-section duct with a constant heat flux under laminar flow conditions.

The fully developed flow through straight uniform square duct has not been studied so far neither for couple stress fluid nor for Jeffrey fluid. The aim of present paper is to investigate the flow of couple stress fluid and Jeffrey fluid in a straight duct of uniform square cross section separately. The channel is filled with homogeneous porous medium. The Cartesian coordinate system has been considered. We find the numerical solution of the governing partial differential equations using finite difference method. The graphical results are presented for velocity profile with various involved fluid parameters for both problems.

2. Formulation of the problem

Consider the steady flow of an incompressible non-Newtonian fluid through a straight square porous duct with uniform square cross-section. We choose the Cartesian coordinate system such that the z-axis along the axis of the duct and x, y-axes along the sides of square duct. Let $x = a$ and $y = a$ be the lengths of the square duct in x and y directions respectively. The fluid is set into motion by a constant pressure gradient in the positive z-direction so that the flow occurs only in z-direction (see Fig. 1). Since the flow is along the z-direction, the velocity at any point in the flow field is expected to be in the form $\bar{q} = (0, 0, w(x, y))$. The equations governing the flow of an incompressible non-Newtonian fluid through the porous medium are given by Nadeem and Akram (2010) and Tripathi (2002)