Journal of the Association of Arab Universities for Basic and Applied Sciences (2015) xxx, xxx–xxx

University of Bahrain

Journal of the Association of Arab Universities for Basic and Applied Sciences

www.elsevier.com/locate/jaaubas
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ORIGINAL ARTICLE

Approximate solutions for solving the Klein–Gordon and sine-Gordon equations

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Received 20 August 2015; revised 3 October 2015; accepted 11 October 2015

KEYWORDS

Variational homotopy perturbation method; Klein–Gordon equation; sine-Gordon equation

Abstract In this paper, we practiced relatively new, analytical method known as the variational homotopy perturbation method for solving Klein–Gordon and sine-Gordon equations. To present the present method's effectiveness many examples are given. In this study, we compare numerical results with the exact solutions, the Adomian decomposition method (ADM), the variational iteration method (VIM), homotopy perturbation method (HPM), modified Adomian decomposition method (MADM), and differential transform method (DTM). The results reveal that the VHPM is very effective.

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1. Introduction

Differential equations can model many physics and engineering problems especially the nonlinear differential equations, but to reach exact solutions is not an easy way. Therefore analytical methods have been used to find approximate solutions. In recent years, many analytical methods such as Variational iteration method (He, 1999b; Muhammad and Syed, 2008; Zayed and Abdel Rahman, 2010b), and modified variational iteration method (Zayed and Abdel Rahman, 2009, 2010a) and variational homotopy perturbation method (Fadhil et al., 2015; Matinfar et al., 2010; Matinfar and Ghasemi, 2010) have been utilized to solve linear and nonlinear equations. The Klein–Gordon and sine-Gordon equations are a two non-linear hyperbolic partial differential equations, which are model problems in classical and quantum mechanics, solitons, and condensed matter physics.

Let us consider the Klein–Gordon and sine-Gordon equation, respectively,

$$u_{tt} - u_{xx} + b_1 u + G(u) = f(x,t), \tag{1}$$

and

$$u_{tt} - u_{xx} + \alpha \sin(u) = f(x,t), \tag{2}$$

subject to initial conditions

$$u(x,0) = f(x), \quad u_t(x,0) = g(x). \tag{3}$$

where u is a function of x and t, G is a nonlinear function, f is a known analytic function, and b_1, α are constants.

In recent years, there has appeared an ever increasing interest of scientist and engineers in analytical techniques for

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Peer review under responsibility of University of Bahrain.

http://dx.doi.org/10.1016/j.jaubas.2015.10.003
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Please cite this article in press as: Yousif, M.A., Mahmood, B.A. Approximate solutions for solving the Klein–Gordon and sine-Gordon equations. Journal of the Association of Arab Universities for Basic and Applied Sciences (2015), http://dx.doi.org/10.1016/j.jaubas.2015.10.003

studying Klein–Gordon and sine-Gordon equations. The Klein–Gordon equation was solved by many analytical methods such as variational iteration method (Elcin, 2008; Semary and Hassan, 2015) differential transform method (Ravi Kanth and Aruna, 2009), the decomposition method (El-Sayed, 2003), homotopy perturbation method (Chowdhury and Hashim, 2009), new homotopy perturbation method (Biazar and Mostafa, 2014), Local fractional series expansion method (Yang et al., 2014), and the tanh and the sine-cosine methods (Wazwaz, 2005). Also, the sine-Gordon equation has been solved analytically by modified decomposition method (Kaya, 2003), variational iteration method (Batiha, 2007; Abbasbandy, 2007), and differential transform method (Biazar and Mohammadi, 2010). Recently many authors studied Klein/sine-Gordon equations (see Fukang et al., 2015; Chen et al., 2014; Dehghan et al., 2015).

The goal of the this paper is to solve Klein–Gordon and sine-Gordon equations by applying another powerful analytical method, called the variational homotopy perturbation method (VHPM), the method is a coupling of homotopy perturbation method (He, 1999a) and variational iteration method (He, 1999b). VHPM was first envisioned by Muhammad and Syed (see Muhammad and Syed, 2008). VHPM used to solve many equations such as Higher Dimensional Initial Boundary Value Problems (Muhammad and Syed, 2008), Benjamin-Bona-Mahony (Fadhil et al., 2015), Zakharove-Kuznetsov equations (Matinfar and Ghasemi, 2010), Fishers equation (Matinfar et al., 2010), the fractional equations (Guo and Mei, 2011), and fractional diffusion equation (Guo et al., 2013).

2. Variational Homotopy perturbation Method (VHPM)

To clarify the basic ideas of VHPM, we consider the following differential equation

$$Lu + Nu = g(x,t). \tag{4}$$

where L is a linear operator defined by $L = \frac{\partial^m}{\partial t^m}$, $m \in \mathbb{N}$, N is a nonlinear operator and $g(x,t)$ is a known analytic function. According to VIM, we can write down a correction functional as follows:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(Lu_n(x,\tau) + N\tilde{u}_n(x,\tau) - g(x,\tau))\, d\tau. \tag{5}$$

where λ is a Lagrange multiplier, which can be identified optimally via a variational iteration method. The subscripts n denote the th approximation, $\tilde{u}$ is considered as a restricted variation. That is, $\delta\tilde{u} = 0$;. Now, we apply the homotopy perturbation method,

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = u_0(x,t) + p\int_0^t \lambda\left(\sum_{n=0}^{\infty} p^n L u_n(x,\tau) + N\sum_{n=0}^{\infty} p^n \tilde{u}_n(x,\tau) - g(x,\tau)\right) d\tau. \tag{6}$$

As we see, the procedure is formulated by the coupling of variational iteration method and homotopy perturbation method. A comparison of like powers of p gives solutions of various orders.

3. Applications

To illustrate the effectiveness of the present method, several test examples are considered in this section.

3.1. Example 1

We consider the linear Klein–Gordon equation

$$u_{tt} - u_{xx} = u, \tag{7}$$

subject to the initial conditions

$$u(x,0) = 1 + \sin(x), \quad u_t(x,0) = 0. \tag{8}$$

According Eq. (5), the correction functional is given by

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda\big((u_n(x,\tau))_{\tau\tau} - (\tilde{u}_n(x,\tau))_{xx} - \tilde{u}_n(x,\tau)\big)\, d\tau. \tag{9}$$

Making the above correction functional stationary, and noting that $\delta\tilde{u} = 0$, we get

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta\int_0^t \lambda\big((u_n(x,\tau))_{\tau\tau} - (\tilde{u}_n(x,\tau))_{xx} - \tilde{u}_n(x,\tau)\big)\, d\tau. \tag{10}$$

which yields the following stationary conditions:

$$\lambda'' - \lambda = 0, \tag{11}$$

$$1 - \lambda'|_{\tau=t} = 0, \tag{12}$$

$$\lambda|_{\tau=t} = 0. \tag{13}$$

Then, the Lagrange multiplier, can be identified as

$$\lambda(\tau) = \sinh(\tau - t). \tag{14}$$

As a result and Eq. (9), we get

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \sinh(\tau - t)\big((u_n(x,\tau))_{\tau\tau} - (u_n(x,\tau))_{xx} - u_n(x,\tau)\big)\, d\tau. \tag{15}$$

Applying the variational homotopy perturbation method, we have

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = u(x,0) + tu_t(x,0) + p\int_0^t \sinh(\tau - t)\left(\left(\sum_{n=0}^{\infty} p^n u_n(x,\tau)\right)_{\tau\tau} - \left(\sum_{n=0}^{\infty} p^n u_n(x,\tau)\right)_{xx} - \sum_{n=0}^{\infty} p^n u_n(x,\tau)\right) d\tau. \tag{16}$$

Comparing the coefficient of like powers of p, we have

$$p^0 : u_0(x,t) = u(x,0) + tu_t(x,0), \tag{17}$$

$$p^1 : u_1(x,t) = \int_0^t \sinh(\tau - t)(u_{0\tau\tau}(x,\tau) - u_{0xx}(x,\tau) - u_0(x,\tau))d\tau. \tag{18}$$

$$\vdots$$

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