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On the limitations of linear growth rates in triply diffusive convection in porous medium

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Abstract The present paper purports to deal with the problem of triply diffusive convection in sparsely distributed porous medium using the Darcy-Brinkman model. Bounds are derived for the modulus of the complex growth rate p of an arbitrary oscillatory perturbation of growing amplitude, neutral or unstable for this configuration of relevance in oceanography, geophysics as well as in many engineering applications. These bounds are obtained by deriving the integral estimates for the various physical quantities by exploiting the coupling between them in the governing equations; and are important especially when at least one boundary is rigid so that exact solutions in the closed form are not obtainable. It is further proved that the result obtain herein is uniformly valid for any combination of rigid and dynamically free boundaries.

1. Introduction

Research on convective fluid motion in porous media under the simultaneous action of a uniform vertical temperature gradient and a gravitationally opposite uniform vertical concentration gradient (known as double diffusive convection) has been an area of great activity due to its importance in the predication of ground water movement in aquifers, in assessing the effectiveness of fibrous materials, in engineering geology and in nuclear engineering. Double diffusive convection is now well known. For a broad view of the subject one may refer to Vafai (2005), Nield and Bezan (2006), Murray and Chen (1989), Chamkha et al. (2002), Umavathi et al. (2005), Zeeshan and Ellahi (2013), Ellahi et al. (2013, 2015), Rashidi et al. (2014, 2015), Hassan and Rashidi (2014), Zeeshan et al. (2014).

Although the double-diffusive convection in porous and nonporous medium is still an active research domain (Swamy, 2014; Choudhary et al., 2015; Khan and Sultan, 2015; Nield et al., 2015; Slim, 2014; Yang et al., 2015; Babu et al., 2014; Chamkha and Al-Naser, 2001; Magyari and Chamkha, 2008; Chamkha et al., 2010; Ellahi et al., 2012; Sheikholeslami et al., 2014a,b; Sheikholeslami and Ellahi, 2015), there are many physical configurations in which more than two diffusing components are present. For example Degens et al. (1973) reported that the saline waters of geothermally heated Lake Kivu are strongly stratified by temperature and salinity which is the sum of comparable concentrations of many salts, while the sea-water contains many salts in concentrations slightly less than the sodium chloride concentration.

The subject of systems having more than two components in porous and nonporous medium has attracted many

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researchers (Griffiths, 1979a; Poulikakos, 1985; Rudraiah and Vortmeyer, 1982; Lopez et al., 1990; Tracey, 1998; Prakash et al., 2015a,b; Ryzhkov and Shevtsova, 2009; Rionero, 2013; Zhao et al., 2014) due to its importance in the study of geothermally heated lakes, earth core, solidification of molten alloys, underground water flow, acid rain effects, natural phenomena such as contaminant transport, warming of stratosphere and magmas and their laboratory models and sea water etc. Some fundamental differences between the double and triply diffusive convection are noticed by these researchers. Among these differences one is that if the gradients of two of the stratifying agencies are held fixed, then three critical values of the Rayleigh number of the third agency are sometimes required to specify the linear stability criteria (in double diffusive convection only one critical Rayleigh number is required). Another is that the onset of convection may occur via a quassi periodic bifurcation from the motionless basic state.

The presence of more than two components in a fluid, each influencing the density and having different diffusive properties, can lead to convective instabilities, often well before a fluid system would become statically unstable. It is now well established that (Griffiths, 1979a,b; Terrones, 1993) the small concentration of a third component with a smaller mass diffusivity can have a significant effect upon the nature of instability; and 'diffusive convection' (oscillatory modes) and direct 'salt finger' modes (steady convection) may simultaneously exist under a wide range of conditions, even if the over-all density stratification is gravitationally stable. Thus, since instability in triply diffusive configuration may occur in the form of oscillatory motions, the problem of deriving the upper limits for the linear growth rate of an arbitrary neutral or unstable oscillatory disturbance of growing amplitude in triply diffusive convection has its own importance in fluid dynamics, especially when at least one of the boundaries is rigid so that exact solutions in the closed form are not derivable as was possible for the cases treated by Griffiths (1979a), Poulikakos (1985) and Rudraiah and Vortmeyer (1982). Banerjee et al. (1981) formulated a novel way of combining the governing equations and the boundary conditions for double diffusive convection problem so that a semicircle theorem is derivable and which in turn yields the desired bounds. Their method has been used to derive the desired bounds for triply diffusive convection in porous medium. Further the result for double diffusive convection in porous medium also follows as a consequence.

In the present paper we have studied triply diffusive convection in a sparsely distributed porous medium by using Darcy-Brinkman model. Darcy flow model is relevant only to densely packed, low permeability porous medium. Darcy's law cannot account for the no-slip boundary condition at the interface of a porous medium and a solid boundary (Kaviany, 1995). Also, experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters indicate that most of the experimental data do not agree with the theoretical predictions based on the Darcy flow model. The Brinkman (1947) extension of the Darcy's law gets around the obstacle by adding a viscous like term to the equations. Givler and Altobelli (1994) have demonstrated that for high permeability porous media the effective viscosity is about ten times the fluid viscosity. Therefore, the effect of viscosities on the stability analysis is of practical interest.

2. Mathematical formulation

A viscous finitely heat conducting Boussinesq fluid layer, saturating a porous medium, of infinite horizontal extension is statically confined between two horizontal boundaries $z=0$ and $z=d$ which are respectively maintained at uniform temperatures T_0 and $T_1(<T_0)$ and uniform concentrations S_{10}, S_{20} and $S_{11}\ (<S_{10}), S_{21}\ (<S_{20})$ (as shown in Fig. 1). It is assumed that the saturating fluid and the porous layer are incompressible and that the porous medium is a constant porosity medium. It is further assumed that the cross-diffusion effects of the stratifying agencies can be neglected. The Brinkman extended Darcy model has been used to investigate the triple diffusive convection in porous medium.

The basic hydrodynamic equations that govern the problem are given by Vafai (2005).

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \tag{1}$$

Equations of motion

$$\frac{1}{\epsilon}\frac{\partial u}{\partial t}+\frac{1}{\epsilon^2}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right) = -\frac{\partial}{\partial x}\left(\frac{\check{p}}{\rho_0}\right)-\frac{\nu}{k_1}u+\frac{\mu_e}{\rho_0}\nabla^2 u, \tag{2}$$

$$\frac{1}{\epsilon}\frac{\partial v}{\partial t}+\frac{1}{\epsilon^2}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right) = -\frac{\partial}{\partial y}\left(\frac{\check{p}}{\rho_0}\right)-\frac{\nu}{k_1}v+\frac{\mu_e}{\rho_0}\nabla^2 v, \tag{3}$$

$$\frac{1}{\epsilon}\frac{\partial w}{\partial t}+\frac{1}{\epsilon^2}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right) = -\frac{\partial}{\partial z}\left(\frac{\check{p}}{\rho_0}\right)-\frac{\nu}{k_1}w+\frac{\mu_e}{\rho_0}\nabla^2 w-\frac{\rho}{\rho_0}g. \tag{4}$$

Equation of heat conduction

$$E\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\kappa\nabla^2 T. \tag{5}$$

Equation of mass diffusion

$$E_1\frac{\partial S_1}{\partial t}+u\frac{\partial S_1}{\partial x}+v\frac{\partial S_1}{\partial y}+w\frac{\partial S_1}{\partial z}=\kappa_1\nabla^2 S_1, \tag{6}$$

$$E_2\frac{\partial S_2}{\partial t}+u\frac{\partial S_2}{\partial x}+v\frac{\partial S_2}{\partial y}+w\frac{\partial S_2}{\partial z}=\kappa_2\nabla^2 S_2. \tag{7}$$

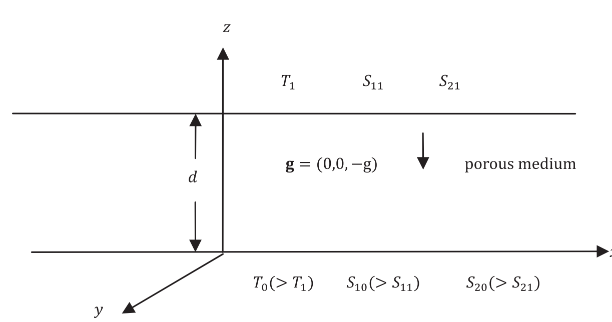

Figure 1 Physical configuration.

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