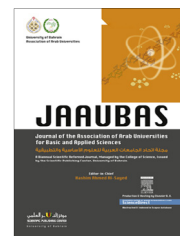




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ORIGINAL ARTICLE

A fractional model of fluid flow through porous media with mean capillary pressure

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Abstract In this paper, we discuss a fractional model arising in flow of two incompatible liquids through homogenous porous media with mean capillary pressure. The solution is derived by the application of the Sumudu transform and the Fourier sine transform. The results are received in compact and graceful forms in terms of the generalized Mittag-Leffler function, which are suitable for numerical computation. The mathematical formulation leads to generalized fractional derivative which has been solved by using a numerical technique by employing the iterative process with the help of appropriate boundary conditions. This problem has great importance in petroleum technology.

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1. Introduction

A porous medium is a material containing pores (voids). Voids are usually filled with a fluid as liquid gas. A porous medium is most often characterized by its porosity. The skeletal portion of the material is often called the matrix or frame. Other properties of the medium such as permeability, electrical conductivity and tensile strength can also be consequent for the respective properties of its constituents (solid matrix and fluid) and the media porosity and pore structure, but these are generally complex. For a poroelastic medium the concept of porosity is usually uncomplicated. This concept of porous media is

used in many areas of applied science and engineering. The oil–water movement in a porous medium is an important problem of petroleum technology and water hydrology (Scheidegger, 1966). Here we consider the injection of water into an oil formation in porous medium providing a two phase liquid–liquid flow problem. Such a problem is generally encountered in secondary recovery process. A number of research workers have also studied phenomenon of flow of two incompatible liquids through homogenous porous media with mean capillary pressure by using different mathematical resources (Bravo and Araujo, 2008; Brooks and Corey, 1964; Corey, 1954; Scheidegger, 1960; Scheidegger and Johnson, 1961). The fractional calculus has gained importance and popularity during the recent years or so, mainly due to its demonstrated applications in science and engineering. For example, these equations are increasingly used to model problems in fluid flow, theology, diffusion, relaxation, oscillation, anomalous diffusion, reaction–diffusion, turbulence, diffusive transport akin to

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diffusion, electric networks, polymer physics, chemical physics, electrochemistry of corrosion, relaxation processes in complex systems, propagation of seismic waves, dynamical processes in self-similar and porous structures and many other physical processes (Hilfer, 2000; Srivastava et al., 2012; Moustafa and Salem, 2006; Podlubny, 1999; He, 1998; Chaurasia and Singh, 2010). Many authors have proposed various methods to handle linear and non-linear fractional differential equations which are of great importance in scientific and technological fields. Among these are differential transform method (He, 1998; Atangana and Alabaraoye, 2013; Atangana and Kilicman, 2013), homotopy perturbation method (Liu et al., 2014), and variational iteration method (He and Wu, 2007).

In this article, we study a fractional partial differential equation associated with the generalized fractional derivative which is governed by the flow of immiscible phases in a homogenous porous medium with initial and boundary conditions. The solution of the fractional model is obtained by using Sumudu and Sine transforms.

2. Preliminary results

The Sumudu transform of a function $f(t)$, determined for all real numbers $t \geq 0$, is the function $F_s(u)$, defined by Watugala (1993), Weerakoon (1994), Asiru (2001), and Belgacem and Karaballi (2005).

$$S\{f(t)\} = \bar{F}(u) = G(u) = \int_0^\infty (1/u)e^{-t/u}f(t)dt. \tag{1}$$

We will also use the following outcome hold by Chaurasia and Singh (2011) as:

$$S^{-1}[u^{\gamma-1}(1-\omega u^\beta)^{-\delta}] = t^{\gamma-1}E_{\beta,\gamma}^\delta(\omega t^\beta). \tag{2}$$

The Fourier sine transform is defined by Debnath (1995).

$$\bar{F}(s, t) = \frac{2}{\sqrt{\pi}} \int_0^\infty f(x, t) \sin sx \, dx. \tag{3}$$

The error function of x is defined by Rainville (1960)

$$erf(x) = \frac{2}{\pi} \int_0^x \exp(-z^2) \, dz \tag{4}$$

and the complimentary error function of x is defined as:

$$erfc(x) = \frac{2}{\pi} \int_x^\infty \exp(-z^2) \, dz \tag{5}$$

A generalization of the Mittag-Leffler function by Mittag-Leffler (1903, 1905)

$$E_\alpha(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma(n\alpha + 1)}, \quad (\alpha \in C, \Re(\alpha) > 0) \tag{6}$$

was introduced (Wiman, 1905) in the general form

$$E_{\alpha,\beta}(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma(n\alpha + \beta)}, \quad (\alpha, \beta \in C, \Re(\alpha) > 0) \tag{7}$$

also derived (Shukla and Prajapati, 2007) in the following integral:

$$\int_0^\infty e^{-st} t^{\beta-1} \frac{d^k}{dz^k} E_{\alpha,\beta}(xt^\alpha) dt = \frac{k!s^{\alpha-\beta}}{(s^\alpha - x)^{k+1}}. \tag{8}$$

The fractional derivative of order $\alpha > 0$ is presented (Caputo, 1967) in the form:

$${}_0^c D_x^\alpha f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{f^{(m)}(\tau)}{(x-\tau)^{\alpha-m+1}} d\tau, \quad m-1 < \alpha < m$$

$$= \frac{d^m f(x)}{dx^m}, \quad \text{if } \alpha = m; m \in N \tag{9}$$

where $\frac{d^m f(x)}{dx^m}$ is the m^{th} derivative of order m of the function $f(x)$ with respect to x . The Sumudu transform of this derivative is given (Chaurasia and Singh, 2010) as:

$$S[{}_0^c D_x^\alpha f(x); s] = u^{-\alpha} \bar{f}(s) - \sum_{k=0}^{m-1} u^{-\alpha+k} f^{(k)}(0+), \quad m-1 < \alpha \leq m. \tag{10}$$

A generalization of the Caputo fractional derivative operator Eq. (9) is given (Hilfer, 2000), by introducing a right-sided fractional derivative operator of two parameters of order $0 < \alpha < 1$ and $0 \leq \beta \leq 1$ as:

$${}_0 D_{a+}^{\alpha,\beta} f(x) = I_{a+}^{\beta(1-\alpha)} \frac{d}{dx} \left(I_{a+}^{(1-\beta)(1-\alpha)} f(x) \right). \tag{11}$$

If we put $\beta = 1$, Eq. (11) reduces the Caputo fractional derivative operator assigned from Eq. (9).

Sumudu transform formula for this operator is given by Hilfer (2000), Belgacem et al. (2003), we hold:

$$S[{}_0 D_x^{\alpha,\beta} f(x); s] = u^{-\alpha} \bar{f}(s) - u^{-\beta(\alpha-1)+1} I_{0+}^{(1-\beta)(1-\alpha)} f(0+); \quad 0 < \alpha \leq 1, \tag{12}$$

where the initial value term

$$I_{0+}^{(1-\beta)(1-\alpha)} f(0+), \tag{13}$$

involves the Riemann–Liouville fractional integral operator of order $(1-\beta)(1-\alpha)$ evaluated in the limit as $x \rightarrow 0+$. For more details and properties of this operator see Tomovski et al. (2010).

The simplest Wright function is defined (Erdélyi et al., 1981; Srivastava et al., 2012) as:

$$\phi(\alpha, \beta; z) = \sum_{k=0}^\infty \frac{1}{\Gamma(\alpha k + \beta)} \frac{z^k}{k!}, \quad \text{where } \alpha, \beta, z \in C, \tag{14}$$

and the general Wright function is defined as:

$${}_p \psi_q(z) = {}_p \psi_q \left[\begin{matrix} (a_i, \alpha_i)_{(1,p)} \\ (b_j, \beta_j)_{(1,q)} \end{matrix} \middle| z \right] = \sum_{k=0}^\infty \frac{\prod_{i=1}^p \Gamma(a_i + \alpha_i k)}{\prod_{j=1}^q \Gamma(b_j + \beta_j k)} \frac{z^k}{k!}, \tag{15}$$

where $z, a_i, b_j \in C$ and $\alpha_i, \beta_j \in R (i = 1, 2, \dots, p$ and $j = 1, 2, \dots, q)$ then Eq. (15) reduces to familiar generalized hyper-geometric function as (Thomas and George, 2006)

$${}_p F_q(a_1, \dots, a_p; b_1, \dots, b_q; z) = \sum_{k=0}^\infty \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \frac{z^k}{k!}. \tag{16}$$

The generalized Navier–Stokes equations are given as (Moustafa and Salem, 2006)

$$\mathcal{W}(\alpha, \beta; z) = \sum_{k=0}^\infty \frac{1}{\Gamma(\alpha k + \beta)} \frac{z^k}{k!}, \quad \text{where } \alpha, \beta, z \in C. \tag{17}$$

The relationship between the Wright function and the complementary Error function is given as

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