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# Dynamic simulation of planetary gear with tooth root crack in ring gear

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ABSTRACT

Planetary gear is widely used in different areas due to its advantages such as compactness, large torque-to-weight ratio, large transmission ratios, reduced noise and vibrations. However, the tooth faults like cracks are seldom concentrated. In this paper, a mesh stiffness model of internal gear pair with a tooth root crack in the ring gear is derived based on the potential energy principle. The mesh stiffness model is incorporated into the dynamic model of a one-stage planetary gear set with 21-degree-of-freedom (DOF) to investigate the effect of the internal gear tooth root crack. The crack cases with different dimensions are designed in this paper to demonstrate their influences on the mesh stiffness and the dynamic performance of the planetary gear set. The simulated results show that bigger reduction in mesh stiffness is caused by the growth in the crack size. And the impulsive vibrations and sidebands can be observed in the dynamic response of the planetary gear set in time and frequency domains, respectively. Both their amplitudes increase as the crack propagation which supply the possibility for them to be the indicators in the condition monitoring and fault diagnosis of planetary gear system.



## 1. Introduction

Due to the advantages such as compactness, large torque-to-weight ratio, large transmission ratios, reduced noise and vibrations, planetary gears are widely used in the industry, national defense and aerospace. That is the reason why there has been numerous research work focusing on the planetary gear transmission system in recent decades. The factors influencing the vibration- and noise-related dynamic responses of planetary gear system can be categorized into two groups: the inherited internal (e.g. mesh stiffness, transmission errors, etc.) and external (e.g. fluctuations of load or speed, etc.) time-varying factors. In these factors, the inherited parameters of the planetary gear were highly investigated due to their determinative roles in the performances. So, the research topics on the analysis of time-varying mesh stiffness [1–4], load sharing among planet gears [5–7], natural modes [8,9], transmission errors [10,11], mesh phasing [12–14], spacing [9,15], and backlash-related nonlinear dynamics [16–18] were mainly concentrated in the previous published documentations. However, fewer published literatures are available about the investigations on planetary gear system with tooth fault.

When the planetary gear transmission systems operate under some inappropriate conditions, such as inadequate lubrication, poor specifications, material defects, and improper manufacturing or installation, it is likely to cause the gear tooth faults. Once the tooth faults appear, the dynamic performances of the gear systems is bound to be affected, which produces undesirable changes in the dynamic behavior, even the serious reduction in the service life of the planetary system [19,20]. However, the changes of the gear dynamic behavior due to the fault supply the opportunity to detect and extract the fault

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feature signals, which is the reason why the studies on gear tooth fault have attracted so much attention. Gear tooth root crack, as one of the frequently encountered gear failure modes, has also been concerned intensively. A series of research work on the crack propagation path have been carried out by NASA [21–25]. It was found that the crack propagation paths depend on the backup ratio which is defined as the ratio of rim thickness to tooth height and it appears to be smooth, continuous, and rather straight with only a slight curvature. The initial inclination crack angle is also a determinative factor for the propagation. For the gear with high backup ratio and big inclination angle, gear tooth root crack would propagate in the tooth body along tooth width. While for those with low backup ratio or small inclination angle, the crack would propagate in the rim. In this paper, only the former case where the crack propagates in the tooth body is investigated. Some other researches about gear tooth crack have been reviewed in our previous paper [26]. In recent decades, few papers studied the effects of the gear tooth faults on the dynamics of planetary gear system due to its structural complexity.

The effect of the tooth surface wear on the dynamic performance of a typical planetary gear set was investigated by Yuksel and Kahraman [27] with employing a computational model. The effects of the crack in the planet carrier plate were carried on by Sparis and Vachtsevanos [28] using experimental data from ground and aircraft tests and by Sunyoung et al. [29] in terms of metallurgical and finite element analysis. Chaari et al. [20] simulated the influence of tooth pitting and cracking on the dynamic responses of a planetary gear by changing the phase and amplitude of the square waveform mesh stiffness. The dynamic responses of a differential planetary system due to the combination of backlash and tooth defects were studied by Wu et al. [30]. Dhanasekaran et al. [31] analyzed crack failure on sun gear of a planetary gear in detail and finally drew a conclusion that the crack occurred due to fatigue. Clegg [32] applied a metallurgical analysis on the failures of the planetary pinion in the earth moving equipment, which indicated that failure was caused by the internal rupture of the gear. In all, there are few studies on the effect of the gear tooth crack on the dynamic performances of planetary gears, especially for that in the ring gear tooth which is the concerned in this paper.

The presence of the gear tooth root crack always results in a reduction in the mesh stiffness [33,34]. Mesh stiffness of health gear pair was computed analytically by Weber [35], Cornell [36]. Kasuba and Evans [37] applied a digitization approach to calculate the mesh stiffness. Yang and Lin [38] calculated the mesh stiffness of a spur gear pair based on the potential energy method. And their model was further refined by Tian [39] and Wu et al. [40] by taking the shear mesh stiffness into consideration. And it was further refined by Chen and Shao [26] with taking the fillet-foundation deflections into account. While for the mesh stiffness calculation of internal gear pairs, the analytical models are seldom found. Hidaka et al. [41] calculated the deformation of the ring gear tooth along line of action with finite element method and analytical model which is based on the work done by Karas [42]. They evaluated the deflection of the ring gear tooth due to the bending moment, the shearing force and the Hertzian contact by replacing the tooth profile by a trapezoid approximately. In this paper, the mesh stiffness model of the internal gear pair is developed based on the potential energy method.

The main objective of this paper is to develop the mesh stiffness model of internal gear pairs with and/or without a crack at the ring gear tooth root based on the potential energy principle. In addition, the crack model developed in our previous paper [26], which can simulate the propagation along both tooth width and crack depth, is adopted in this paper to investigate the effect of the crack on the mesh stiffness of the internal gear pair and on the dynamic responses of the planetary gear system. This is expected to supply the theoretical information for the condition monitoring and fault diagnosis of planetary gear system.

This paper consists of five parts: Reviews on the research topics about the mesh stiffness calculation and dynamic responses of the planetary gear set with or without tooth root crack is carried out in the first segment where the motivation to form this paper is introduced. Then, the mesh stiffness model of the internal gear pair with tooth root crack in the ring gear is derived in the second segment. Based on this mesh stiffness model, the effect of the tooth root crack on the mesh stiffness of the internal gear pair is investigated in Section 3 and its influence on the dynamic responses of the planetary gear set is analyzed in Section 4 which is followed by the conclusions in Section 5.

## 2. Mesh stiffness of internal gear pair with tooth root crack

The derivation of the calculation of the mesh stiffness of an internal gear pair with tooth root crack is mainly carried on in this section based on the potential energy principle. The schematic of the ring gear tooth with crack is displayed in Fig. 1. The crack model in this paper is limited to the crack propagation only in the ring gear tooth body but not in the rim and the ring gear rim is assumed to be rigid.

The potential energy principle, saying the work done by the external forces is equal to the potential energy stored in the deformation of the flexible body, has been used broadly in the calculation of mesh stiffness of external gears with or without tooth fault [38–40]. In this paper, the stiffness of the internal gear tooth is derived with the potential energy method by regarding the internal gear tooth as a non-uniform cantilever beam. Under the action of the external applied force, the work done by it along the line of action due to the bending, shear and axial compressive deformations of the internal tooth can be obtained as [26,38,39],

$$U_b = \frac{F_m^2}{2K_b}, \quad U_s = \frac{F_m^2}{2K_s}, \quad U_a = \frac{F_m^2}{2K_a}, \tag{1-a, b, c}$$