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Full Length Article

An analytical method for solving exact solutions of a nonlinear evolution equation describing the dynamics of ionic currents along microtubules

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Abstract

In this article, a variety of solitary wave solutions are observed for microtubules (MTs). We approach the problem by treating the solutions as nonlinear RLC transmission lines and then find exact solutions of Nonlinear Evolution Equations (NLEEs) involving parameters of special interest in nanobiosciences and biophysics. We determine hyperbolic, trigonometric, rational and exponential function solutions and obtain soliton-like pulse solutions for these equations. A comparative study against other methods demonstrates the validity of the technique that we developed and demonstrates that our method provides additional solutions. Finally, using suitable parameter values, we plot 2D and 3D graphics of the exact solutions that we observed using our method.

Keywords: Analytical method; Exact solutions; Nonlinear evolution equations (NLEEs) of microtubules; Nonlinear RLC transmission lines

1. Introduction

Many technical difficulties in developing an understanding of nonlinear phenomena occur in various fields of engineering, mathematical physics, and applied mathematics, such as hydrodynamics, optical fibres, fluid mechanics, biology, plasma physics, solid state physics, chemical systems, and geochemistry. Calculating analytical and numerical solutions, particularly solitary and travelling wave solutions, of nonlinear evolution equations (NLEEs) plays an important role in soliton theory [1]. Recently, it has become attractive to determine exact solutions, analytical solutions and numerical solutions of NLEEs using symbolic programmes, such as Maple, Mathematica and Matlab, that ease intricate and monotonous algebraic computations. Many authors have developed methods to identify exact solutions

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of NLEEs, including the Rational Homotopy Perturbation Method [2], the Jacobi elliptic function method [3–5], the generalized Riccati equation mapping method [6–8], (G'/G)-expansion method [9–13], the Sumudu transform method [14–16], the multiple exp-function algorithm method [17,18], the tanh-function method [19,20], the extended tanh-function method [21], the modified extended tanh-function method [22,23], and the $\exp(-\varphi(\xi))$-expansion method [24–27].

The objective of this work is to apply the last method mentioned, the $\exp(-\varphi(\xi))$-expansion method, to identify exact solutions of nonlinear PDEs of special interest in nanobiosciences, namely, transmission line models of nanoionic currents along microtubules, which have significant roles in cellular signalling. Microtubules (MTs) are cytoskeleton biopolymers shaped as nanotubes that are essential for cell motility, cell division, intracellular trafficking and information processing within neuronal processes. MTs have also been implicated in higher neuronal functions, including memory and the emergence of consciousness. How MTs handle and process electrical information, however, is still unknown. In this study, we establish a new model for ionic waves along MTs based on polyelectrolyte features of cylindrical biopolymers. Each tubulin dimer protein is an electric element with a capacitive, resistive and negative incrementally resistive property [28]. Particular attention was paid in [29,30] to the role of nanopores (NPs) existing between neighbouring dimers within an MT wall, which exhibits properties similar to those of ionic channels. These NPs could be used to explain the behaviour of MTs as biomolecular transistors capable of amplifying electrical information in neurons. The physical details of the derivation of the following equation describing their ionic currents are provided in [29–37]:

$$R_2C_0L^2u_{xxt}+L^2u_{xx}+2R_1C_0\delta uu_t-R_1C_0u_t=0, \tag{1}$$

where $R_1=10^9\,\Omega$ and $R_2=7\times10^6\,\Omega$ stand for the transverse and longitudinal components of resistance of an elementary ring (ER), and the parameter $\delta(\delta<1)$ describes the nonlinearity of an ER capacitor in an MT. In this instance, $L=8\times10^{-9}\,m$, while $C_0=1.8\times10^{-15}\,F$ is the total maximal capacitance of the ER. Zayed et al [6] employed the improved generalized Riccati equation mapping method and applied it to solving a nonlinear partial differential equation describing the dynamics of ionic currents along microtubules and constructing travelling wave solutions. Sekulic et al. [22] investigated the equation of MTs as a nonlinear RLC transmission line to obtain solitary wave solutions by applying the modified extended tanh function (METF) method.

2. Description of the exp(−Φ(ξ))-expansion method

In this section, we briefly describe the major steps of the $\exp(-\varphi(\xi))$-expansion method. Consider a general NLEE of the form

$$P(u,\ u_t,\ u_x,\ u_{xx},\ u_{tt},\ u_{tx},\ldots)=0\ , \tag{2}$$

where $u=u(x,t)$ is an unknown function and P is polynomial in $u=u(x,t)$ and its partial derivatives, in which higher order derivatives and nonlinear terms are involved. To solve eq. (2) by this method, one has to resort to the following steps:

Step 1. To find the travelling wave solution of (2), introduce the wave variable $\xi=x\pm ct$, where $u(x,t)=u(\xi)$. Then

$$\frac{\delta}{\delta t}=-c\frac{\delta}{\delta\xi},\ \frac{\delta^2}{\delta t^2}=c^2\frac{\delta^2}{\delta\xi^2},\ \frac{\delta}{\delta x}=\frac{\delta}{\delta\xi},\ \frac{\delta^2}{\delta x^2}=\frac{\delta^2}{\delta\xi^2}, \tag{3}$$

and so on for other derivatives. With the help of (3), the NLEE (2) changes to an ODE as

$$\Re(u,u',u'',u''',\cdots)=0\,, \tag{4}$$

where u', u'' etc. denote derivatives of u with respect to ξ, and $\Re$ is a polynomial in u. Next, integrate the ODE (4) as many times as is applicable and set the constants of integration to be zero.

Step 2. The solution of (4) can be expressed by a polynomial in $(\exp(-\Phi(\xi))$ as

$$u(\xi)=A_0+A_1\ \exp(-\Phi(\xi))+\sum_{i=2}^{N}A_i(\exp(-\Phi(\xi)))^{i}\ , \tag{5}$$