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Hydromagnetic flow and radiative heat transfer of nanofluid past a vertical plate

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Abstract

Hydromagnetic flow of an incompressible viscous nanofluid past a vertical plate in the presence of thermal radiation is investigated both analytically and numerically. The radiative heat flux is described by the Rosseland diffusion approximation in the energy equation. The governing non-linear partial differential equations are converted into a set of ordinary differential equations by suitable similarity transformations. The resulting ordinary differential equations are successfully solved analytically with the help of homotopy analysis method and numerically by the fourth order Runge–Kutta method with shooting technique. The effects of various physical parameters are analyzed and discussed in graphical and tabular forms. The effects of some physical parameters such as Lewis number, Prandtl number, buoyancy ratio, thermophoresis, Brownian motion, radiation parameter and magnetic parameter are analyzed on the velocity, temperature and solid volume fraction profiles as well as on the reduced Nusselt number and the local Sherwood number. An excellent agreement is observed between present analytical and numerical results.

Keywords: Nanofluid; Hydromagnetic; Homotopy analysis method; Vertical plate and thermal radiation

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1. Introduction

A nanofluid is a new class of heat transfer fluids that contains a base fluid and nanoparticles. The use of additives is a technique applied to enhance the heat transfer performance of base fluids. The nanofluid has many applications. For example, it is used as coolants, lubricants, heat exchangers, micro channel heat sinks and many others [1]. The term 'nanofluid' was first proposed by Choi [2] to indicate engineered colloids composed of nanoparticles dispersed in a

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base fluid. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [3]. The influence of nanoparticles on natural convection boundary layer flow over a vertical plate was considered by Kuznetsov and Nield [4]. Khan and Pop [5] considered the problem of boundary layer flow past a stretching sheet in nanofluids. Bachok et al. [6] investigated the boundary layer flow of a nanofluid over a moving surface in a flowing fluid. Kuznetsov and Nield [7] analyzed the double-diffusive natural convective boundary layer flow of a nanofluid over a vertical plate. Khan and Aziz [8] investigated the boundary layer flow of a nanofluid past a vertical surface with a constant heat flux. Gorla and Chamkha [9] studied the natural convection flow past an isothermal horizontal plate in a porous medium saturated by a nanofluid. Aziz and Khan [10] investigated the natural convective flow of a nanofluid over a convectively heated vertical plate.

Magnetohydrodynamic boundary layer flow is of considerable interest due to its wide usage in industrial technology and geothermal application, high temperature plasmas applicable to nuclear fusion energy conversion, MHD power generation systems and liquid metal fluids. Due to its wide range of applications, the following researchers investigated the magnetic field effect on fluid flow problems in different geometries [11–20]. Yahyazadeh et al. [21] inspected the effect of magnetic field on the free convection flow of a nanofluid over a linear stretching sheet using differential transform method.

The thermal radiation effect may significant at high operating temperatures in engineering processes, under many non-isothermal situations and in situations where convective heat transfer coefficients are small. In view of these applications, the thermal radiation effects on fluid flow were investigated by the followers [22–28].

Keeping this in mind, we investigated the thermal radiation effect on the flow of a nanofluid past a vertical plate in the presence of transverse magnetic field both analytically and numerically. The governing non-dimensional ordinary differential equations are solved analytically using homotopy analysis method and numerically by shooting method.

2. Formulation of the problem

Consider the two-dimensional boundary layer flow of a nanofluid over vertical plate in the presence of magnetic field intensity and the thermal radiation. We select a co-ordinate frame in which the x-axis is aligned vertically upwards. Consider a vertical plate at $y = 0$. At this boundary, the temperature T and the nanoparticle volume fraction ϕ take constant values T_w and ϕ_w respectively. The temperature T and the nanoparticle volume fraction of the nanofluid ϕ take values T_∞ and ϕ_∞ respectively, as $y \to \infty$. We also consider influence of a constant magnetic field strength B_0 which is applied normally to the sheet. It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field. Under the above assumptions, the governing equations for the flow field can be written in dimensional form as [4].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \rho_f \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) - \sigma B_0^2 u + \left[(1 - \phi_\infty) \rho_{f\infty} \beta g (T - T_\infty) - (\rho_p - \rho_{f\infty}) g (\phi - \phi_\infty) \right] \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T - \frac{1}{(\rho c)_f} \left(\frac{\partial q_r}{\partial y} \right) + \tau \left[D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial T}{\partial y} \right)^2 \right] \tag{3}$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

where u and v are the velocity components along the x and y directions respectively. p is the fluid pressure, ρ_f is the density of base fluid, ρ_p is the nanoparticle density, μ is the absolute viscosity of the base fluid, $\alpha = \frac{k}{(\rho c)_f}$ is the thermal diffusivity of the base fluid, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ is the ratio of nanoparticle heat capacity and the base fluid heat capacity, ϕ is the local solid volume fraction of the nanofluid, β is volumetric thermal expansion coefficient of the base fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, T is the local temperature and g is the acceleration due to gravity, B_0 is the constant magnetic field and q_r is the radiative heat flux.

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