

Stratified smooth two-phase flow using the immersed interface method

Petter Andreas Berthelsen *, Tor Ytrehus

*The Fluids Engineering Group, Department of Energy and Process Engineering, Norwegian University of Science and Technology,
Kolbjørn Hejes vei 2, N-7491 Trondheim, Norway*

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Abstract

The immersed interface method is used to derive a numerical method for solving fully developed, stratified smooth two-phase flow in pipes. This sharp interface technique makes the representation of the interface independent of the grid structure, and it allows for using an arbitrary shaped interface. The two-dimensional steady-state axial momentum equation is discretized and solved using a finite difference scheme on a composite, overlapping grid with local grid refinement near the interface and near the pipe wall. A low Reynolds number $k-\epsilon$ turbulence model is adopted to account for the effect of turbulence. A level set function is used to represent the interface. Numerical results are presented for laminar and turbulent flows. The numerical method compares well with analytical solution for laminar flow, and it shows acceptable agreement with experimental data for turbulent flow. A few examples are given to demonstrate the capability of the method to solve flow problems with a complex shaped interface.

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1. Introduction

Stratified two-phase flow occurs frequently in several industrial applications. One example is the transport segment in the petroleum industry, in which correct predictions of pressure gradient and phase fractions are essential to obtain optimal production and transportation rates of oil and gas. Hydrocarbons are often transported in pipelines over great distances, tens of kilometers long. The pipelines often cross hilly terrain, resulting in transition from stratified flow to other less desirable flow regimes, such as slug and intermittent flow. The fluctuating nature of slug flow may cause serious operating problems and lead to an increase in cost due to the necessity of large slug catchers at the outlet of the lines. Therefore, reliable models are of great importance for estimating the correct

pressure gradient and phase fractions in order to predict and prevent unwanted flow behaviour.

Most common models for two-phase pipe flow are based on greatly simplified representation of the flow structure. Early approaches have been partly empirical, using parameters such as proposed by Lockhart and Martinelli [17]. Later, mechanistic models were introduced. These semi-analytical methods use simplified physical models to determine the frictional pressure gradient and in situ phase fractions. Wall and interfacial shear stresses are based on empirical correlations, such as the well-known Blasius formula for single-phase flow [31].

There is a growing interest in exploring the possibilities in using computational fluid dynamics (CFD) in predicting multiphase flow behaviour, or so-called computational multi-fluid dynamics (CMFD¹). Both Volume of Fluid (VOF) methods [14,27] and level set methods have been successfully employed in modelling evolving surfaces [11,30]. However, it should be noticed that these

* Corresponding author. Current address: Centre for Ships and Ocean Structures, Norwegian University of Science and Technology, N-7491 Trondheim, Norway. Tel.: +47 73 55 11 05; fax: +47 73 59 55 28.

E-mail addresses: petter.a.berthelsen@ntnu.no (P.A. Berthelsen), tor.ytrehus@ntnu.no (T. Ytrehus).

¹ This acronym was suggested by Yadigaroglu [32] to become the “brand name” of this promising and continually emerging field of fluid dynamics.

approaches are based on the immersed boundary method by Peskin [26], which smear out discontinuities across the interface and yield poor estimates of interfacial shear stress.

Discontinuous fluid properties lead to non-smooth solutions across the interface. Traditional finite difference methods work poorly for these problems since the numerical discretization is not well-defined across the interface. To properly treat discontinuities at the interface, it is therefore common practice to align the interface along a grid line. When using body-fitted grids, the numerical problem is subdivided into separate single-phase regions with a moving boundary at the interface. The boundary values are interchanged between the different fluid regions to ensure that the interfacial boundary conditions are satisfied. Within each phase, standard and well-behaved numerical techniques are employed to solve the governing equations. But these methods have their practical limitations since transient problems are still extremely computationally expensive to solve in multiple spatial dimensions. Instead, it is necessary to introduce some means of simplification to the mathematical formulation.

For stratified flow in pipes, the flow is commonly assumed to be fully developed so that it is sufficient to only consider a cross-section of the pipe. In previous studies, a two-dimensional body-fitted bipolar coordinate system is used to represent the stratified two-phase pipe flow with a flat interface [13,19,28,20,21]. These two-dimensional methods brought more details into the analysis than the one-dimensional models, although they confined only to two-phase flow with simple interface configurations. For typical gas–liquid flow, the assumption of a flat interface is realistic in most practical cases, but for low liquid loads, or liquid–liquid flow, the interface may take a complex curved shape. One of the disadvantages of using body-fitted grids is the difficulty in representing a complex shaped interface, or even multiple interfaces, without putting too much effort into the grid generation.

The intention of this work is to investigate the possibilities of adopting the decomposed immersed interface method [3] to predict two-dimensional fully developed stratified pipe flow. The immersed interface method was first introduced by Leveque and Li [16] to avoid the complexity of body-fitted grid generators and still maintain a well-defined numerical discretization. In the decomposed approach presented by Berthelsen [3], componentwise correction terms are added to the finite difference stencil in order to make the discretization well-defined across the interface at any position regardless of the numerical grid. A level set function is used to determine the location of the interface. The method is originally developed for Cartesian grids, but is here generalized to any orthogonal coordinate system.

In this approach, the two-dimensional steady-state axial momentum equation is solved for the axial velocity field. For turbulent flow, the eddy viscosity is modelled using a low Reynolds number k – ε model. The principal difference

with the present method compared to other more commonly used body-fitted bipolar coordinate system is the way the boundary conditions at the interface are incorporated into the numerical scheme. This method allows for using arbitrary shaped and multiple interfaces, which makes it possible to also consider three-phase flow [4]. For simplicity, the interface is assumed smooth and non-deformable, since the main objective with this study is only to examine the applicability of the immersed interface method for stratified two-phase flow. The effect of interfacial waves is more a question of proper modelling and can be dealt with at a later stage e.g. [5].

The rest of the paper is organized as follows: The mathematical formulation is discussed in Section 2, and the numerical method is described in Section 3. The method is validated with analytical and experimental results, accompanied by a few numerical examples in Section 4. In Section 5, the summary and conclusions are given.

2. Mathematical model

2.1. Governing equations

In fully developed, incompressible and stratified smooth two-phase pipe flow the axial component of the time-averaged steady-state momentum equation for each phase can be written as

$$\frac{\partial}{\partial x} \left(\mu_e \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_e \frac{\partial u}{\partial y} \right) - \frac{dp}{dz} - \rho g \sin \theta = 0. \quad (1)$$

Here u is the time-averaged axial velocity, ρ is the density, g is the gravitational acceleration, θ is the pipe inclination angle, μ_e is the effective viscosity, and dp/dz is the pressure gradient in the axial direction (Fig. 1).

The effective viscosity μ_e reduces to the molecular viscosity μ if the flow is laminar. Otherwise, if the flow is turbulent, the effective viscosity is given as the sum of the molecular and eddy viscosity, $\mu_e = \mu + \mu_t$. Following a similar approach as Newton and Behnia [20] for turbulent flow, the turbulence is assumed to be isotropic and the eddy viscosity is modelled using the low Reynolds number k – ε model of Lam and Bremhorst [15], i.e.

$$\mu_t = \rho C_\mu f_\mu \frac{k^2}{\varepsilon}.$$

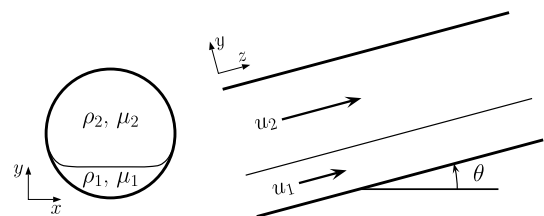


Fig. 1. Illustration of stratified smooth two-phase pipe flow.

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