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Asymptotic analysis of fracture propagation in materials with rotating particles

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ABSTRACT

Cosserat continuum for particulate materials has characteristic lengths commensurate with small particle sizes leading to *small-scale Cosserat continuum* – an asymptotics intermediate between the characteristic length and the crack length. We show that the main asymptotic term can be obtained from the classical crack problem by finding rotations from the displacements and, using the Cosserat constitutive equations finding the moment stresses. We obtained that Mode I, II stress singularities are the same in classical continuum; the moment stress has a stronger singularity (power 3/2). Bond bending and breakage caused by relative particle rotations constitutes the dominant mechanism of crack propagation.

1. Introduction

Materials whose constituents have (or acquire at a certain stage of loading) the ability to rotate cover a wide range of engineering and natural materials including granular materials, heavy fractured parts of the Earth's crust or the rock mass that form blocky structure, heavily fractured coatings or tailings as well as rock, concrete, masonry and ceramics at advanced stages of damage. There is evidence obtained in experiments, field observations and discrete element modelling that the deformation of such materials does involve rotation of their constituents. Microrotations were observed in zones of strain localisation in granular materials in direct physical experiments (e.g. [1–4]) and in discrete element method-type simulations (e.g. [5–7]). A possibility of grain rotation at the fracture surface was considered in [8]. Block rotations were observed in masonry [9], in the Earth's crust in the form of the movement of tectonic blocks (e.g. [10–15]) and in interlocking structures [16,17]. Hereafter we refer to the rotating constituents as '*particles*' and to materials with rotating constituents as *particulate materials*.

Failure in such materials often occurs by propagation of macroscopic fractures even if they appear as shear bands (e.g. [18–20]). Given the presence of considerable rotations in shear bands the question arises what the role of rotations in fracture propagation is. One way to answer this question would be to model fracture propagation both with and without taking rotations into account and compare the results.

Modelling deformation and fracture of particulate materials presents a considerable challenge. While discrete element type modelling of particulate materials is currently the dominant tool in investigating their mechanical behaviour, in the cases when the model scale considerably exceeds the particle dimensions the continuum modelling is still a viable

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Nomenclature

l_{micro}	microscopic length scale of the material
L	half crack length
d	size of the non-elastic zone at the crack tip
H	representative volume element size
u	displacement
σ_{ij}	the stress tensor
μ_{ij}	the moment stress tensor
φ_i	the Cosserat rotation vector
γ_{ij}	the strain tensor
κ_{ij}	the curvature-twist tensor
$\mu,\ \lambda$	Lame constants
$\alpha,\ \beta,\ \gamma,\ \varepsilon$	Cosserat moduli
B	bending modulus
$l,\ l_2$	Cosserat characteristic lengths
k_n	normal stiffness of the binder
k_s	shear stiffness of the binder
k_{φ_n}	twist stiffness of the binder
k_{φ_s}	bending stiffness of the binder
T_1, T_2	coordinate transformations
$\mathbf{b}_i$	Burgers vectors
$\mathbf{\Omega}$	Frank's vector
ρ_i	dislocation density
ρ_φ	disclination density
K_{ij}, M_{ij}	kernels of integral equations
K_I	Mode I stress intensity factor
K_{II}	Mode II stress intensity factor
σ_m	maximum microscopic stress in the binder

alternative, given the sheer number of particles involved and the absence of the detailed information on every individual particle. Continuum modelling represents materials at scales macroscopic with respect to the material microstructure. When classical continua are introduced they do not inherit the microstructural sizes such that the constitutive behaviour is scale-invariant. The emergence of rotational degrees of freedom necessitates the use of the micropolar or Cosserat mechanics [21–25] whose constitutive equations introduce their own length scales. In the elastic Cosserat continuum the internal length scales are consequences of the fact that the elastic moduli relating the moment stress and rotation gradients have units, which differ from the ones of the moduli that relate the stress and displacement gradients or rotations. This difference is a factor of length square, which gives rise to a set of characteristic lengths characterising different pairs of moduli.

Thus in Cosserat continuum modelling one deals with two kinds of characteristic lengths: the microstructural lengths, which are implicit to the theory and the explicit Cosserat lengths, which quantify different resistance of the material to the relative displacements and rotations of the particles. The lengths of both kinds should be taken into account in the continuum modelling of fracture propagation, in particular when the principles of LEFM are used. Indeed, the classical linear elastic fracture mechanics considers an asymptotics of $d/L_{cr} \to 0$, where L_{cr} is the crack/fracture dimensions and d is the size of the process zone – the zone of inelastic deformation at the crack tip. Since the classical elastic continuum is scale invariant (it contains no characteristic length) the stress singularity at the crack/fracture tip is expressed through a power law ('square root singularity') and this is what the machinery of the LEFM is based upon.

The above asymptotics is however not the only one assumed in the LEFM. After all the LEFM is a continuum theory valid as long as the given material can be modelled as a continuum (an *equivalent continuum*). This requires the so-called separation of scales, which presumes that a representative volume element of size, H, can be introduced satisfying the following double inequality (e.g. [26–29]):

$$l_{micro} \ll H \ll L \tag{1}$$

where l_{micro} is the characteristic size of the material microstructure (in our case the largest particle dimension), L is the characteristic length of the variations of the external fields. The role of L can be played either by the crack length, L_{cr} or the size of the process zone, d, if it to be modelled using the continuum. Thus, the process zone models imply that its size $d \gg H \gg l_{micro}$.

Since the equivalent continuum does not refer explicitly to the volume element size H, double inequality (1) implies that continuum modelling presumes the following double asymptotics:

$$\frac{l_{micro}}{H} \to 0, \quad \frac{H}{L} \to 0 \tag{2}$$