Engineering Fracture Mechanics 150 (2015) 19–27

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Technical Note

Effect of initial T-stress on stress intensity factor for a crack in a thin pre-stressed layer

Zi-Cheng Jiang [a], Guo-Jin Tang [a], Xian-Fang Li [b,*]

[a] College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, PR China
[b] School of Civil Engineering, Central South University, Changsha 410075, PR China

ARTICLE INFO

Article history:
Received 9 May 2015
Received in revised form 11 October 2015
Accepted 12 October 2015
Available online 17 October 2015

Keywords:
Initial stress
Pre-stressed structure
Crack
Stress intensity factor
Energy release rate

ABSTRACT

Fracture of a pre-stressed thin elastic layer is studied and modelled as bending of double clamped beams with axial force. Using the beam theory, the strain energy in bending state of the beam with an axial force is derived. Explicit expressions for energy release rate and stress intensity factor (SIF) for a crack with initial transverse stress (T-stress) are obtained. Results show that tensile (compressive) initial T-stress decreases (increases) the SIF and impedes (promotes) crack growth. The influence of compressive initial stress is greater than that of tensile initial stress in the same magnitude.

1. Introduction

Crack problems have attracted much attention in recent decades. It is recognized in linear elastic fracture mechanics that stress intensity factor (SIF) is a significant fracture parameter to describe singular elastic field near a crack tip. As well known, a transverse stress, called T-stress, parallel to the crack plane does not affect the SIF, but influence the elastic field near the crack tip by changing the nonvanishing term of the elastic field [1]. Therefore, a two-parameter fracture criterion based on a *K–T* dominating elastic field is more suitable to predict crack growth. The effect of elastic T-stresses on crack initiation direction and yield zone has been analyzed [2–9].

On the other hand, in civil engineering, many practical structures are applied in an environment of action of an initial stress. This class of structures are sometime called pre-stressed structures. In addition, for composite structures such as fiber-reinforced or particle-reinforced structures, the inhomogeneity of materials may give rise to initial stresses or residual stresses due to change in environment temperature [10]. In particular, more applications of initial stresses can be found in micromechanical devices [11] and biological science [12]. For such pre-stressed structures, the study on crack problems is also of importance. However, relevant work is very limited [13]. Soos [14] analyzed stress concentration in a prestressed elastic plane with a crack and obtained the asymptotic behavior of the incremental elastic fields in the close vicinity of the crack tips. For a penny-shaped crack under radial shear, the influence of initial stress on fracture of a cracked half-space was dealt with using the integral equation method by Nazarenko et al. [15]. Radi et al. further studied the effects of pre-stress on crack-tip fields in an elastic and incompressible solid [16]. For a mode III interface crack in a pre-stressed solid, asymptotic incremental displacement and stress fields in the vicinity of the crack tip were numerical calculated using

* Corresponding author. Tel.: +86 731 8816 7070; fax: +86 731 8557 1736.
E-mail address: xfli@csu.edu.cn (X.-F. Li).

http://dx.doi.org/10.1016/j.engfracmech.2015.10.034
0013-7944/

the complex potential method [17]. Akbarov and Turan used a finite element method to study the influence of initial stresses on the SIF for an orthotropic strip [18]. In addition, Corso et al. also investigated stress concentration near a rigid line inclusion in a prestressed elastic plane [19]. Although some studies on the effect of initial stresses on elastic fields near a crack tip have been conducted, there is no information on an explicit dependence of the SIF on initial stress reported. To fill this gap, in this paper we make an attempt to adopt a simple beam theory to achieve this purpose.

This paper develops the bending model of double clamped beams subjected to axial loading to simulate a cracked elastic layer with initial stress. An emphasis is placed on the determination of the SIF, which sheds light on the influence of initial stress on the SIF.

2. Theoretical model

Consider a cracked elastic layer of thickness $2h$ subjected to an initial transverse stress (T-stress) along the crack plane. The crack is located at $-a < x < a$ and $2a$ denotes the crack length. Under the action of incremental forces or stresses, as shown in Fig. 1, it is much desired to obtain SIFs near the crack tips. We assume that the initial elastic fields induced by the initial stress are small. In this case, in order to determine the incremental elastic fields of a cracked elastic layer, equilibrium equations for the class of problems can be linearized as

$$\frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{13}}{\partial z} + \sigma_{11}^0 \frac{\partial^2 u}{\partial x^2} = 0, \tag{1}$$

$$\frac{\partial \sigma_{13}}{\partial x} + \frac{\partial \sigma_{33}}{\partial z} + \sigma_{11}^0 \frac{\partial^2 w}{\partial x^2} = 0, \tag{2}$$

where u and w are the components of incremental displacement along the x and z directions, respectively, σ_{11}^0 is an initial stress along the x direction, and σ_{ij} is stress tensor. The outline of derivation is given in Appendix A.

Although a detailed analysis of a pre-stressed elastic layer with a crack based on a two-dimensional theory of elasticity is possible, but cumbersome, here we propose a simple theoretical model to obtain a dependence of the SIF on the initial T-stress. Consider the case of a thin elastic layer. Since the thickness of the elastic layer is small enough as compared to the crack length $2a$, we model this crack as double clamped beams. An advantage of this approach is that an explicit expression for the SIF can be derived in terms of initial stress. To do so, let us simplify the stated problem and use the simplest Euler–Bernoulli theory of beams.

For Euler–Bernoulli beams, owing to the hypothesis that the cross-section of the beam remains perpendicular to the neutral surface prior to and posterior to bending deformation, the displacement component u can be represented by the unique independent displacement component w in terms of the following relation

$$u = -z \frac{\partial w}{\partial x}. \tag{3}$$

Thus multiplying (1) by z and then integrating over cross-section, one gets

$$M' - Q - \sigma_{11}^0 I w''' = 0, \tag{4}$$

where the prime denotes differentiation with respect to x, I is the second moment of cross-sectional area, and M and Q stand for bending moment and shear force at cross-section, viz.

$$M = \int_A \sigma_{11} z dA, \quad Q = \int_A \sigma_{13} dA, \tag{5}$$

where A is the cross-sectional area. Additionally, integrating (2) over cross-section leads to

$$Q' + q + \sigma_{11}^0 A w'' = 0, \tag{6}$$

where q is distributed loading. From (4) and (6), we eliminate shear force Q and get

$$M'' + q + P w'' - \sigma_{11}^0 I w^{IV} = 0, \tag{7}$$

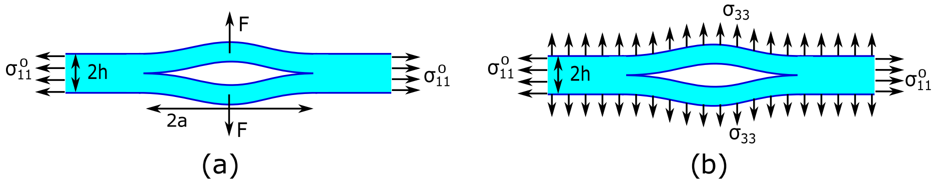

Fig. 1. A cracked elastic layer with an initial stress σ_{11}^0 along the crack plane; (a) case A: opening of crack under a pair of concentrated forces, (b) case B: opening of crack under uniformly distributed loading.