Ultrasonics - Sonochemistry 47 (2018) 75–98

Contents lists available at ScienceDirect

Ultrasonics - Sonochemistry

journal homepage: www.elsevier.com/locate/ultson

A strict formulation of a nonlinear Helmholtz equation for the propagation of sound in bubbly liquids. Part I: Theory and validation at low acoustic pressure amplitudes

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ARTICLE INFO

Keywords:
Ultrasound
Cavitation modeling
Nonlinear acoustics
Bubble dynamics
Nonlinear Helmholtz equation

ABSTRACT

This paper strictly demonstrated a nonlinear Helmholtz equation, with its corresponding new expressions for the wave number of the mixture, for the propagation of sound trough a bubbly liquid. The demonstration was conducted under the assumption of periodicity of volume fluctuations, the acoustic approximation and considering only mono-harmonic pressure oscillations. The model revealed a beautiful symmetry between the average acoustic energy density and the average energy dissipation, as well as between the time average of the first and second derivatives of such fluctuations. The nonlinear model was validated with available experimental data at very low pressure amplitudes yielding the same results as the linear model. However, unlike the linear model, the advantage of the nonlinear model is that the wave number of the mixture is function of the pressure amplitude, which has great implications to model the sound propagation on cavitating bubbly liquids where the linear theory greatly under-predicts.

1. Introduction

The transmission of sound through liquids containing bubbles is strongly attenuated by the presence of those bubbles even if the volume fraction is very low. For instance, bubble volume fractions as low at 0.4% can reduce the speed of sound below the speed of sound in the air contained in the bubbles [1]. This is a well-known fact that has been experimentally confirmed and is well document in the scientific literature. The sound propagation through a liquid medium containing bubbles has been an active area of research since the work of Foldy [2] who considered the bubbles as scatters interacting with the incident acoustic field. The propagation of the sound field thorough a bubbly liquid is completely specified if the wave number of the “effective medium” is known [3]. Foldy’s model determined the effective wave vector for a monodispersed population of bubbles as a function of the speed of sound in the pure liquid, the angular frequency, the population of bubbles per unit of volume and the scattering function for a single bubble of a specific radius. The main assumption on Foldy’s model is that the scattering function is composed of the contribution of the incident and scattered fields but neglects the interaction between scatters. Hence, Foldy’s model is restricted to low volume fractions where the separation between neighboring bubbles is large [3]. The scattering function is estimated as a function of the resonance frequency of the bubble and the dampening constants due to thermal, viscous and radiative energy loses. The calculation of the dampening is due to the model of Prosperetti [4] who determined dampening mechanisms after linearizing the radial oscillations of bubbles induced by a harmonic incident pressure field. By linearizing the oscillation of bubbles, the model is restricted not only to low volume fractions but also to very low pressure amplitudes wherein the bubble radius is assumed to change proportionally to the pressure field.

A second approach to model the sound propagation through a bubbly liquid is a continuum approach. A theory of waves in bubbly liquids was proposed from a set of nonlinear equations, based on physical reasoning, by van Wijngaarden [5,6]. This was done by a semi-empirical volume averaging of the bubbly liquid mass and momentum conservation equations which are closed by a Rayleigh-Plesset equation. Subsequently, Caflish et al. [7] validated van Wijngaarden approach by developing a model that reduces to the equations of van Wijngaarden. Caflish’s model is a mathematically rigorous derivation of average equations described from the microscopic motion of liquids and bubbles. Caflish’s model neglected convection, which is valid at slow velocities, and assumed a very small fraction of bubbles. Caflish equations are nonlinear and therefore computationally demanding to solve.

Commander and Prosperetti [8] developed a linear model for both mono and poly-disperse bubble populations. It dramatically simplified the nonlinear complexities of Caflish equations. This linearized model allows modelling sound waves via a Helmholtz equation in which a

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https://doi.org/10.1016/j.ultsonch.2018.04.014
Received 2 September 2017; Received in revised form 22 April 2018; Accepted 25 April 2018
Available online 30 April 2018
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complex wave number accounts for the effects of the mixture containing bubbles of different sizes. The model of Commander and Prosperetti can predict the pressure field distribution of acoustic waves moving on bubbly liquids at frequencies lower than resonance, low bubble volume fractions and low pressure amplitudes where linear simplifications are valid.

Linear models over predict the sound attenuation at resonant conditions. Furthermore, the sound attenuation at higher frequencies (well above resonance) is still unclear. Some works have been devoted to account for the interaction of bubbles to explain the over prediction at resonance. For instance, Fuster, Conoir and Colonius [9] simulated the effect of bubble–bubble interactions on the acoustic properties of the effective medium by modifying the linear theory accounting for a critical characteristic distance at which bubble–bubble interactions affect attenuation. Ando, Colonius and Brennen [10] modified the dispersion relation by including the effect of liquid compressibility on the natural frequency of bubbles, since the wavelength of ultrasonic waves may be similar or shorter than the mean bubble spacing. Kargl [3] also argued that the linear theory of Commander and Prosperetti can be improved if the Keller's bubble dynamics equation is expressed for bubbles in the effective medium. A recent improvement of the linear model, was proposed by Zhang and Du [11], who performed a surface integral over the bubble surface to account for non-uniform pressure fields occurring for large values of kR_o (wave number times equilibrium radius). The model was validated using Kol'tsova et al. [12] experimental data showing that this new approach improved predictions at large values of kR_o. Recently, Fuster and Montel [13], whose work was then extended by Zhang at al [14,15], generalized the classical linear theory of diluted bubbly liquids by including the vapour mass transfer from the liquid boundary into the bubble, showing that mass transfer effects play an important role in the phase speed and attenuation at frequencies below the bubble resonant frequency.

Linear models have no dependency on pressure amplitude rendering those models inadequate to represent acoustic pressure fields on cavitating bubbly liquids, where the acoustic pressure amplitude is higher than the Blake threshold causing the bubble size to increases many fold to then abruptly collapse and rebound as shown by single bubble sonoluminescence (SBSL) experiments. Nonlinear models, such as the one of Vanhille and Campos-Pozuelo [16] shows that for low pressure amplitude, bubbles oscillate harmonically and proportionally to the acoustic pressure, but at high pressure amplitudes nonlinear distortions on the bubble oscillations are predicted with a strong asymmetry between the compression and rarefaction phases. Furthermore, in cavating bubbly liquids sound is attenuated many fold higher than predictions from the linear model; the linear theory predicts aberrantly low attenuations compared to experimental data [17]. Addressing this problem Louisnard [18] proposed a simple model that coupled the behaviour of inertial bubbles, represented by the Rayleigh Plesset equation (RPE), with the acoustic field, represented by a nonlinear Helmholtz equation with a complex wave number. Louisnard exactly correlated the imaginary part of the square of the wave number with the thermal and viscous energy dissipation of bubbles.

The method outlined by Louisnard allowed coupling the oscillations of single bubbles with the pressure field by accounting for the nonlinear behaviour of inertial bubbles, which are subjected to high pressure amplitudes, instead of assuming small bubble-oscillations around equilibrium, as assumed in the linear model of Commander and Prosperetti [8], which is valid at low pressure amplitudes only. Louisnard calculation of the energy dissipation terms agreed with the linear model for low pressure amplitudes, well below the Blake threshold, but it departed from the linear theory when the pressure amplitude approached and exceeded the Blake threshold. For pressure amplitudes above that threshold the dissipation terms were several orders of magnitude higher than the linear prediction showing that the linearization of Commander and Prosperetti [8] does not represent correctly the strong attenuation observed on cavitating systems. Jamshidi and Brenner [19] extended Louisnard's approach by using the Keller–Miksis Equation (KME), instead of the classical Rayleigh Plesset equation (RPE), to account for liquid compressibility. Hence, their model accounted for acoustic radiation as a third dissipation mechanism.

One of the strong features of Louisnard model is that it correlates the dynamics of strongly attenuating inertial bubble with a nonlinear Helmholtz equation. Dogan and Popov [20] compared the non-linear model of Louisnard, calculating the dissipation terms as per Jamshidi and Bernner [19] approach, with the linear model of Commander and Prosperetti. They found that pressure amplitudes calculated by the linear theory are approximately 4 times larger than calculations by the nonlinear approach showing that attenuation is significantly underestimated by the linear theory. Dogan and Popov's simulations also demonstrated that the nonlinear model captured more realistically the spatial distribution of the cavitation zones.

In spite of the great progress achieved by Louisnard to model sound waves moving through cavitating liquids, the model is not complete from a theoretical point of view because of two factors: firstly, the real part of the square of the wave number was approached to the linear model of Commander and Prosperetti [8], and secondly, the nonlinear Helmholtz equation used by Louisnard was demonstrated from the linear theory. Hence, the model of Louisnard is a heuristic combination of the nonlinear oscillations of bubble with the linear theory. In this study the work of Louisnard was advanced by firstly formulating an exact correlation of the real part of the square of the wave number, and secondly, by strictly demonstrating a nonlinear Helmholtz equation, that accounts for the effect of pressure amplitude, without relying upon the linear theory and producing a mathematically complete and self-contained nonlinear model.

Initially an inhomogeneous Helmholtz equation, with a real wave number, is formulated from the Caflish equations. Then, the imaginary part of the square of a mixture wave number, demonstrated by Louisnard [18], was combined with its real part, strictly demonstrated in this article, to finally recast the inhomogeneous equation into a nonlinear Helmholtz equation that accounts for pressure amplitude. The equation is valid under three simplifications: firstly, the acoustic approximation, where density and speed of sound in the liquid are assumed constant; secondly, assuming that bubbles oscillate periodically with a period corresponding to the wave period; and thirdly, harmonics and oscillatory pressure components at frequencies different than the primary frequency were neglected. In this first part of a two parts paper the nonlinear model is formulated and validated with the experimental data of Silberman [21], Kol'tsova et al. [12] and Wilson, Roy and Carey [22]. The nonlinear model yielded almost the same predictions of the linear theory when run at very low pressure amplitudes. The nonlinear model has the advantage of accounting for the effect of pressure on attenuation representing the strong attenuation observed on cavitating bubbly liquids more realistically than the linear model. The main contribution of this paper is on the demonstrations on appendixes F, G, H, J, K and L but to facilitate the reading of the paper, for those that are not interested on mathematical demonstrations, section 2 "Mathematical model" explains the mathematical framework and background, including the main findings from the appendixes but without demonstrations. The accompanied paper part II applies the nonlinear model to acoustic cavitation.

2. Mathematical model

2.1. Bubble dynamics

Simplified bubble dynamic models are second order ordinary differential equations representing the oscillation of gas bubbles in a liquid under the action of acoustic fields. These models are based on the classical Rayleigh-Plesset equation (RPE) [23–26] describing the radial oscillations experienced by bubbles due to an external acoustic pressure p:

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