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Multi-parameter fracture criteria for the estimation of crack propagation direction applied to a mixed-mode geometry

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ABSTRACT

A parametric study has been performed on an eccentric asymmetric four-point bending specimen in order to investigate initial crack propagation direction. Under various levels of mixed-mode conditions, the generalized form of the maximum tangential stress criterion and the strain energy density criterion has been derived by means of the multi-parameter description of the crack-tip stress field. A discussion on the need to consider several more initial terms of the Williams expansion during fracture behaviour assessment is presented and some recommendations are stated based on the results of the analyses.

1. Introduction

The fracture behaviour of a large group of structural materials can be described by means of the classical linear elastic one-parameter fracture mechanics concept. In this case, the approximation of the near-crack-tip stress field is performed using the stress intensity factor (SIF) as the single-controlling parameter. Generally, this conception works under the assumption that the zone around the crack tip with nonlinear/plastic behaviour is very small in comparison to the typical structural dimensions; see e.g. [1] or other textbooks on fracture mechanics.

On the other hand, if the mutual proportions of the nonlinear zone extent and the size of the investigated cracked body are comparable, the fracture process can no longer be expected to be brittle in nature. This situation is typically more likely for quasi-brittle or elastic–plastic materials/structures. Fracture processes occurring in these cases are more complicated and many works have been concerned with developing approaches that will allow their precise description. Recently, attention has been given to those works which have shown that it is reasonable to take into account more parameters for stress field approximation than merely the stress intensity factor, as is usual; see e.g. [2–11]. The introduced multi-parameter approach is based on the Williams solution of the crack-tip stress field distribution in a cracked specimen, see [12]. The authors' works, see e.g. [13–18], as well as others, show that using more terms of the series expansion enables the crack-tip stress field to be expressed more effectively than if only the first term, equivalent to the stress intensity factor, is considered.

Moreover, it has been shown in some recent works (see e.g. [19] or [20]) that the *T*-stress (the second term of the Williams expansion) can have a significant influence on the crack propagation path. In Ref. [21] the author of this paper also presents some studies of the crack path using the multi-parameter (generalized) maximum tangential stress criterion; however, only one cracked configuration is investigated. The Refs. [19–21] as well as the more advanced work [22] dealing

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Nomenclature

a	crack length
A_n	coefficients of terms of the Williams expansion for mode I of loading
B_m	coefficients of terms of the Williams expansion for mode II of loading
d	half span between the applied forces in the analysed test geometry
E	Young's modulus
F	applied force
$f_{\sigma_{ij}}$	known functions of the Williams expansion corresponding to loading mode I and to component σ_{ij} of the stress tensor
$g_{\sigma_{ij}}$	known functions of the Williams expansion corresponding to loading mode II and to component σ_{ij} of the stress tensor
f_{u_i}	known functions of the Williams expansion corresponding to loading mode I and to component u_i of the displacement vector
g_{u_i}	known functions of the Williams expansion corresponding to loading mode II and to component u_i of the displacement vector
κ	Kolosov's constant ($\kappa = (3-\nu)/(1+\nu)$ for plane stress and $\kappa = 3 - 4\nu$ for plane strain)
$K_{\mathrm{I}},\ K_{\mathrm{II}}$	stress intensity factors corresponding to mode I and mode II of loading, respectively
L	length of specimen
$n,\ m$	indices of the individual terms of the Williams power series describing mode I and mode II of loading, respectively
$N,\ M$	numbers of terms of the power series considered for stress state approximation (N – mode I, M – mode II)
$r,\ \theta$	polar coordinates
S	crack eccentricity
u_i	components of the displacement vector
W	specimen width
$x,\ y$	Cartesian coordinates
α	relative crack length ($\alpha = a/W$)
γ	initial crack propagation angle, see Fig. 1
γ_{norm}	normalized initial crack propagation angle, i.e. the ratio of the value calculated from the multi-parameter fracture criterion to the value obtained from numerical analysis
μ	shear modulus
ν	Poisson's ratio
σ_{ij}	stress tensor components ($i,j \in \{x,y\}$)
$\sigma_{\theta\theta}$	tangential stress
Σ	strain energy density
BCM	boundary collocation method
EA4PB	eccentric asymmetric four point bending
FE	finite elements
FEM	finite element method
HCE	hybrid crack element method
LS	local symmetry
MTS	maximum tangential stress
ODM	over-deterministic method
SED	strain energy density
SIF	stress intensity factor
WE	Williams expansion

with X-FEM simulations of fatigue crack growth demonstrate the importance of the multi-parameter fracture mechanics for one of the basic fracture mechanics tasks, prediction of the crack path.

Because of all the reasons described above, in this paper two most often used fracture criteria for estimation of the crack propagation direction are investigated; their multi-parameter/generalized form is suggested and its accuracy/efficiency is tested within a parametric study carried out on a mixed-mode geometry. The main focus is on the investigation of initial crack propagation direction and one of the important aspects of this paper is its large extent: the kink angle is determined for a wide range of mode-mixity situations.

It should be also mentioned that the multi-parameter form of the stress field description can bring another benefits, such as better dealing with the well-known size/geometry/boundary effect [9,11,23–28]. Nevertheless, it is not contained in this paper; this work is focused on the importance of the higher-order terms of the Williams expansion for specific conditions which is demonstrated via an investigation of the initial crack propagation direction performed on a mixed-mode geometry. The results and conclusions should support and extend the ongoing research on the application of this multi-parameter approach in more advanced fracture mechanics tasks.