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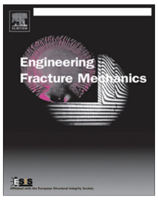

3-D stress intensity factors due to full autofrettage for inner radial or coplanar crack arrays and ring cracks in a spherical pressure vessel

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ABSTRACT

Three dimensional, stress intensity factor for inner radial or coplanar crack arrays and ring cracks prevailing in an autofrettaged spherical pressure vessel are evaluated. The 3-D analysis is performed via the finite element method implementing a novel realistic autofrettage residual stress field incorporating the Bauschinger effect. SIFs are evaluated for arrays of radial or coplanar cracks and ring cracks of depth to wall thickness ratios of $\boldsymbol{a/t} = 0.1–0.6$, and ellipticities of $\boldsymbol{a/c} = 0.2–1.0$ in fully autofrettaged spherical vessels, $\varepsilon = 100\%$, of $\boldsymbol{R_o}/\boldsymbol{R_i} = 1.1, 1.2,$ and 1.7. SIFs are evaluated for radial arrays containing $\boldsymbol{n} = 1–20$ cracks, and for arrays of coplanar cracks of $\delta = 0–0.95$ densities.

1. Introduction

More than one hundred years ago, the process of autofrettage was suggested by Jacob of the French artillery [1] for the purpose of increasing the allowable pressure in gun barrels, thus extending their firing range. Later, it was found that the autofrettage process has an additional substantial benefit in decreasing the vessel susceptibility to cracking, i.e., delaying crack initiation and slowing down crack growth rate, hence considerably increasing the total fatigue life of the barrel. Autofrettage has been further developed and has been widely used for cylindrical pressure vessels in a variety of industries for more than a century.

Spherical pressure vessels, though less common than cylindrical ones, are widely used in industry mainly due to their optimal specific strength (strength/weight) and their ease of packing. Spherical pressure vessels are used, for example, as propellant/oxidizer/pneumatic tanks on space-crafts and aircraft, storage tanks for pressurized chemical substances, gas tanks on LNG (liquefied natural gas) carriers, cookers for the food industry, and as containment structures in nuclear power plants. Moreover, whenever ***extremely high pressure*** occurs, such as in high explosion containment tanks or in the apparatus used to manufacture artificial diamonds and other crystals, spherical pressure vessels are practically the only feasible solution.

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Nomenclature

a	crack depth
c	crack half length
K_I	mode I SIF
K_I^{Ring}	mode I SIF for a ring crack
K_{Imax}	maximum SIF along crack front
K_{IA}	mode I SIF due to autofrettage
K_{IAmax}	maximum SIF due to autofrettage along crack front
K_{IN}	combined SIF
K_{IP}	mode I SIF due to internal pressure
K_0	normalizing SIF (Eq. (1))
K_{00}	normalizing SIF, $K_{00} = -\sigma_y\sqrt{R_i}$
N	number of fatigue cycles
n	number of cracks in the array
Q	shape factor for lunular or crescentic crack (Eq. (2))
p	internal pressure
R_i	inner radius of the spherical vessel
R_o	outer radius of the spherical vessel
r, θ, φ	spherical coordinates
t	spherical vessel's wall thickness

Greek symbols

β	angle defined in Fig. 1c
θ	angle defined in Fig. 1c
$\boldsymbol{\varepsilon}$	level of autofrettage
ν	Poisson's ratio
$\boldsymbol{\delta}$	crack density defined as $\boldsymbol{\delta} = \boldsymbol{\beta}/\boldsymbol{\theta}$ (see Fig. 1c).
σ_y	initial yield stress
$\boldsymbol{\psi}$	parametric angle for lunular and crescentic cracks (Fig. 1e and f)
$\boldsymbol{\psi}_0$	value of $\boldsymbol{\psi}$ at the cusp – the intersection of the crack front and the inner surface of the vessel

Acronyms

DOF	degrees of freedom
FEM	finite element method
LEFM	linear elastic fracture mechanics
SIF	stress intensity factor

Some of these spherical pressure vessels are manufactured from a series of double curved petals welded along their meridional lines [2], and some are composed of two hemispheres manufactured by: press forming, direct machining, machining of forgings, or by spin-forming. The two hemispheres are joined together by conventional, TIG (Tungsten inert gas), or EB (electron beam) girth weld on the equatorial plane. Both types of these vessels are susceptible to cracking along the welds due to one or more of the following factors: cyclic pressurization–depressurization, the existence of a heat-affected zones near the welds, tensile residual stresses within this region, and the presence of corrosive agents. As a result, one or more radial (Fig. 1b) or coplanar cracks (Fig. 1c) develop from the inner surface of the vessel on the respective welding planes. In certain cases the coplanar cracks on the equatorial plane coalesce becoming one inner ring crack (Fig. 1d).

To date, autofrettage is rarely applied to spherical pressure vessels and the possible beneficial effect on such vessels has hardly been investigated. Perl and Berenshtein [3–5] have evaluated, for the first time, a large number of 3-D SIFs due to internal pressure for arrays of radial and coplanar cracks of various lunular[3], crescentic[4] and ring shapes, prevailing at the inner surface spherical vessels of various geometries. Furthermore, Perl et al. [6] recently evaluated numerous 3-D SIFs due to autofrettage for a single inner radial/coplanar crack in an overstrained spherical vessel. It is worthwhile noting that the little empirical evidence available to the authors at present, point to the fact that inner lunular/crescentic cracks develop in spherical pressure vessels, rather than in semi-elliptical ones. However, no experimental data is available to corroborate whether these crack geometries are maintained during crack growth.

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[3] A lunular crack is defined as a planar, part-through crack, whose shape is enclosed by two circular arcs of different radii, one concave and one convex, which intersect at two points, having an ellipticity of $\boldsymbol{a/c} = 1$ (Fig. 1e).

[4] A Crescentic crack is defined as a planar, part-through crack whose shape is enclosed by two intersecting arcs, the concave one which is elliptical, and the convex one which is circular, having an ellipticity of $\boldsymbol{a/c} \neq 1$ (Fig. 1f).