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Weight function for an edge-cracked rectangular plate

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ABSTRACT

In-situ edge-cracked rectangular plates (width H, length L) comprised of first-year sea ice on McMurdo Sound were tested during recent field trips to Antarctica, and expressions applicable for a wide range of crack lengths are sought for the stress-intensity-factor (SIF), and crack-opening-displacement (COD), given that the edge-crack is subjected to arbitrary crack-face loading. A weight function able to provide the required accurate wide-ranging expressions for an edge-cracked rectangular plate (ECRP) subject to arbitrary crack-face loading is developed in this paper, given $H/L = 0.25, 0.5, 1.0, 1.5, 2.0$, and 4.0. The accuracy of the ECRP weight function is assessed by making comparisons with the edge-cracked strip subjected to pure bending (using $H/L = 4.0$), using the essentially identical SIF and CMOD results independently obtained by Kaya and Erdogan (1987) and Bakker (1995). Comparisons are also made with the SIF values obtained by Fett (1999) for the ECRP subjected to pure bending (for $H/L = 1.0, 1.5$, and 2.0). The double-cantilever-beam (DCB) study in Foote and Buchwald (1985) is used to assess the accuracy of the ECRP SIF for the case of concentrated loads at the crack mouth, for $H/L = 1/4$ and $H/L = 1/2$. For the same loading and $H/L = 4$, the study by Kaya and Erdogan (1980) of an edge-cracked infinite strip is used to assess accuracy. For crack-face concentrated loading acting at $X/L = y_i$, given $H/L = 4$, the accuracy of the ECRP SIF weight function is assessed by examining associated data reported by Kaya and Erdogan (1978).

1. Introduction

In-situ edge-cracked rectangular plates comprised of first-year sea ice on McMurdo Sound were tested during recent field trips to Antarctica, and expressions applicable for a wide range of crack lengths are sought for the stress-intensity-factor (SIF), crack-opening-displacement (COD), T-stress [1–3], the B-stress ($B = T\sqrt{\pi A}/K(A)$) [4], and crack opening area (COA). A weight function [5,6] able to provide the required accurate wide-ranging expressions for an edge-cracked rectangular plate (ECRP) subject to arbitrary loading is developed in this paper. The procedure described in [7] is followed, modified by analytical knowledge [3,8,9] of the limiting behavior as $A \to 0$ and $A \to L$ (see Fig. 1). Relevant studies in the literature include [10–13]. The weight function in [10] does not include the reference crack-mouth-opening-displacement (CMOD) as an input condition, and therefore the expressions quickly lose accuracy for $A/L > 0.5$, as expected [14].

In this paper, the weight function for the ECRP is developed. While a previous paper [8] describes the approach in more detail, this presentation is self-contained. Then the reference solutions for the ECRP are discussed, including an assessment of their accuracy. Next in this paper,the ECRP T-stress and B-stress values are examined. Finally, to assess the accuracy of the

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Nomenclature

a	A/L
A	crack length
B	$T\sqrt{\pi A}/K(A)$
COA	crack-opening-area
COD	crack-opening-displacement
CMOD	crack-mouth-opening-displacement
DCB	double cantilever beam
ECRP	edge-cracked rectangular plate
E	Young's modulus
E'	E plane stress; $E/(1-\nu^2)$ plane strain
f_r	coefficient of the reference SIF
H	width of the rectangular plate
H_r	weight function
K	Mode I SIF induced by Σ
K_r	reference SIF
L	length of the plate parallel to the crack
SIF	stress-intensity-factor
X	distance from the crack mouth on the crack plane $Y = 0^{\pm}$
T	T-stress
U	half of COD for arbitrary crack face loading Σ
$\mathcal{U}$	Green's function for U
U_r	half of COD for uniform crack face loading
ν	Poisson's ratio
σ	uniform crack face pressure on $0 \leqslant X \leqslant A, Y = 0^{\pm}$
Σ	arbitrary crack face pressure on $0 \leqslant X \leqslant A, Y = 0^{\pm}$

ECRP weight function developed in this paper, the weight function is used to examine the pure bending of edge-cracked rectangular plates as well as the case of edge-cracked rectangular plates subjected to equal and opposite concentrated loads acting at the crack mouth.

2. The weight function method

The purpose of the present paper is to provide accurate wide-ranging SIF and COD expressions for the ECRP. To this end, consider the ECRP configuration portrayed in Fig. 1; the shape of the body is symmetric with respect to the crack line. The crack face pressure $\Sigma(X)$ is also assumed to be symmetric with respect to the crack line. A versatile method to determine the sought-after expressions is the weight function method [5–8,18]. Knowledge of a two-dimensional elastic crack solution (hereafter called the reference solution) as a function of crack length A for any reference crack face pressure $\Sigma_r(X)$ enables one to determine the stress intensity factor $K(A)$ and COD $U(A,X)$ for the same body under arbitrary loading $\Sigma(X)$:

$$\begin{aligned} K(A) &= \int_0^A \Sigma(X)H_r(A,X)dX, \quad H_r(A,X) = \frac{E'}{K_r(A)}\frac{\partial U_r}{\partial A}(A,X) \\ E'U(A,X) &= \int_X^A K(T)H_r(T,X)dT = \int_0^A \Sigma(S)\mathcal{U}(A,X,S)dS \\ H_r(T,X) &= \frac{1}{\sqrt{2\pi T}}\sum_{i=1}^{5}\mathcal{G}_i\left(\frac{T}{L}\right)\left(1-\frac{X}{T}\right)^{i-\frac{3}{2}}, \quad \mathcal{U}(A,X,S) = \int_{\max(S,X)}^{A} H_r(T,X)H_r(T,S)dT \end{aligned} \tag{1}$$

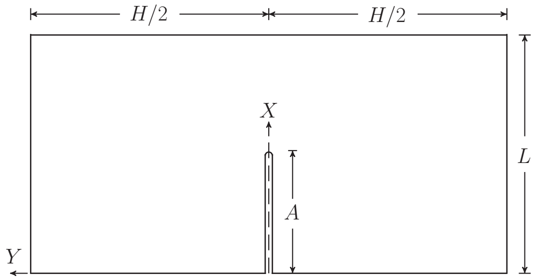

Fig. 1. A single edge crack in a rectangular plate of width H, length L, crack length A.