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Smoothing the propagation of smeared cracks

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ABSTRACT

This work presents a new local crack tracking technique to improve numerical simulations of localized fracture processes in quasi-brittle structures. The algorithm primarily focuses on reduction of mesh-induced directional bias by the determination of smoothly curved C^1 – continuous crack paths within and across conventional continuum elements with quadratic displacement fields. The algorithm further enables to postpone the moment of crack path fixation.

Combined with a classical smeared crack model, the proposed crack propagation algorithm is validated by sequentially linear analyses on tensile and mixed-mode plain concrete fracture tests. Specifically for quadratic elements the results show an increase of the mesh objectivity, and more realistic load–displacement and cracking behaviors are obtained.

1. Introduction

Predictions on ultimate load capacities, failure mechanisms and post-peak behaviors of quasi-brittle structures with nonlinear finite element analysis can be impeded by mesh-induced directional bias. This type of mesh dependency has especially been observed when using approaches in the standard smeared crack concept as isotropic damage models or the fixed and rotating smeared crack models, e.g. [9,26,43]. More advanced models as the embedded crack model (E-FEM) or Strong Discontinuity Approach (SDA) and the eXtended Finite Element Method (X-FEM) are less sensitive to this lack of mesh objectivity, due to their constitutive and kinematic enrichments, e.g. [4,5,24,30,35]. However, these enrichments might not be the only reason. Generally, aforementioned advanced material models are also equipped with a crack tracking algorithm, e.g. [1,35,52,54]. The addition of such an algorithm appears even to be inevitable to capture localized deformation patterns properly. When using E-FEM without a crack tracking technique, similar directional mesh bias compared to approaches in the standard smeared crack concept has been observed [31]. And also within the framework of X-FEM the level set method as tracking technique has become a key ingredient for an accurate description of the crack propagation process. So, if enhancements with crack tracking techniques are indispensable for these advanced models, it might be at least useful for approaches in the standard smeared crack concept as well.

Regarding crack tracking, the main principle of this technique is to trace and designate potential crack paths within an arbitrary finite element (FE) discretization [37]. Elements crossed by a crack path are allowed to damage, while the others are restrained from that, keeping their constitutive relation linearly elastic. The determination of the crack propagation direction should be in line with the adopted failure criterion. For instance, once the maximum principal stress exceeds

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Nomenclature

$d_{crit;crk}$	damage threshold value for crack path fixation
d_e	maximum damage level in an element
d_j	damage values in integration point j, $j = 1, \ldots, n_{ip}$
$\mathbf{e}$	normalized vector or propagation path direction at exit point
E_0	Young's modulus
f_t	tensile strength
G_f	fracture energy
l	crack band width
L_i	periodic length scales, $i = x, y$
n_{cn}	number of corner nodes
n_{ip}	number of integration points in an element
$\mathbf{n}_j$	normal of potential crack in integration point j, $j = 1, \ldots, n_{ip}$
$\mathbf{N}$	coefficient matrix with evaluated shape functions
N_i	standard shape function in corner node i, $i = 1, \ldots, n_{cn}$
$\mathbf{N_{inp}}$	coefficient matrix with evaluated shape functions at input positions
r_{excl}	user-defined exclusion radius
$\mathbf{r}_i$	normalized propagation vector in corner node i, $i = 1, \ldots, n_{cn}$, added for regularization
$\mathbf{R}^k$	standard rotation matrices, $k = 1, 2$
$\mathbf{s}$	normalized start vector or propagation path direction at entry point
S_{j1}, S_{j2}	opposite boundary edges/faces, $j = x, y, c$
$\mathbf{t}_j$	normalized propagation vector in integration point j, $j = 1, \ldots, n_{ip}$
$\mathbf{t}^k$	normalized orthogonal to maximum principal strain direction, $k = 1, 2$
$\widehat{\mathbf{T}}(\xi, \eta)$	estimated normalized propagation vector field
u_{cr}	total inelastic displacement
$\mathbf{w}$	weight factors
w_{ult}	ultimate crack opening at which no stresses can be transferred anymore
$\mathbf{W}$	diagonal matrix that contains weight factors
θ	mesh orientation for the test with periodic boundary conditions
θ_j	potential crack normal angle in integration point j, $j = 1, \ldots, n_{ip}$
ν	Poisson's ratio
$\widehat{\boldsymbol{\varphi}}(\xi, \eta)$	estimated scalar propagation direction field
$\widehat{\boldsymbol{\varphi}}_{\mathbf{cn}}$	estimated propagation directions in corner nodes
$\overline{\boldsymbol{\varphi}}$	input propagation directions

Acronyms

CMSD	crack mouth sliding displacement
CPA	crack propagation algorithm
DEN	double-edge-notched
E-FEM	Elemental enrichment Finite Element Method
ep	exit point
FE	finite element
ip	integration point
cn	corner node
SDA	Strong Discontinuity Approach
SEN	single-edge-notched
SLA	Sequentially Linear Analysis
sp	starting point
X-FEM	eXtended Finite Element Method

the tensile strength in a quasi-brittle material as concrete, it is a natural choice to define the crack propagation direction perpendicular to this maximum principal tensile stress direction. Other crack propagation criteria are reported in [16,38].

Besides aforementioned level set method, the following types of crack tracking algorithms can be distinguished: global [37], non-local [17] and local strategies [55]. The terminology *global*, *non-local* and *local* indicates the domain where information is obtained from in order to predict the crack path propagation. Each type has its own advantages and disadvantages [17,23,38]. Generally, the trend is that a wider considered domain increases the complexity of implementation and the computational costs, but also the robustness.

Within the standard smeared crack concept, and particularly in the framework of continuum damage models, some of the above mentioned algorithms have been successfully applied [8,20]. The enhanced damage models showed significant reduc-