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Stress fields induced by a non-uniform displacement discontinuity in an elastic half plane

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ABSTRACT

This paper presents the exact closed-form solutions for the stress fields induced by a two-dimensional (2D) non-uniform displacement discontinuity (DD) of finite length in an isotropic elastic half plane. The relative displacement across the DD varies quadratically. We employ the complex potential-function method to first determine the Green's stress fields induced by a concentrated force and then apply Betti's reciprocal theorem to obtain the Green's displacement fields due to concentrated DD. By taking the derivative of the Green's functions and integrating along the DD, we derive the exact closed-form solutions of the stress fields for a quadratic DD. The solutions are applied to analyze the stress fields near a horizontal DD in the half plane with three different profiles: uniform (constant), linear, and quadratic. The same methodology is applied to an inclined normal fault to investigate the effect of different DD profiles on the maximum shear stress in the half plane as well as on the normal and shear stresses along the fault. Numerical results demonstrate considerable influence of the DD profile on the stress distribution around the discontinuity.

1. Introduction

Scientists and engineers study cracks for many reasons. Two key reasons are to (1) understand the associated stress concentrations or singularity features, and (2) accurately predict the life span of cracked structures or media.

Numerical methods such as the finite element method (FEM) and boundary element method (BEM) have been utilized by many researchers to solve crack problems. The BEM based on displacement discontinuity (DD) has been proved to be particularly efficient [1–3]. The indirect BEM also has been used to treat single and multiple displacement discontinuities (DDs) in 2D finite and infinite regions [4] and to calculate stress intensity factors at crack tips in 2D anisotropic elastic solids [5]. An accurate single-domain BEM for 2D infinite, finite, and semi-infinite anisotropic solids [6] has been extended to three-dimensional (3D) anisotropic media [7]. The Riemann–Hilbert method can be adopted to solve 2D crack problems in an infinite, homogeneous, anisotropic plate [8]. A general higher-order DD method coupled with an indirect BEM has been applied to the quasi-static analyses of radial cracks produced by blasting [9]. Complex crack problems such as multiple branched and intersecting cracks also have been investigated using the numerical manifold method [10], which also has been applied to 2D

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Nomenclature

Latin

a_j, b_j, c_j	constants to determine the relative displacement discontinuity profile in j-direction
A, B	complex constants
$f_1()$, $f_2()$, $f_3()$	function of
F_x, F_y	line force component along x- and y-direction respectively
i	imaginary unit
$k_1, k_2, \ldots, k_{15}$	complex variables
L	length of displacement discontinuity
$\boldsymbol{n}$	normal vector
n_p	p-component of normal vector
p, q, r, s, w	dummy complex variables
t	parameter that varies between 0 and 1 along the displacement discontinuity
t_m	location along displacement discontinuity between 0 and 1 at which relative displacement is known; $0 < t_m < 1$
u_k	displacement component in k-direction
$u_{k,\,j}$	derivative of k-component of displacement with respect to coordinate j
$\Delta\boldsymbol{u}$	relative displacement discontinuity vector
Δu_{j1}, Δu_{j2}	relative displacement discontinuities along j-direction at starting and ending points of displacement discontinuity
Δu_{jm}	relative displacement discontinuities along j-direction at $t = t_m$
Δu_q	relative displacement discontinuities along q-direction
x, y	coordinates of the field point of the line force
x_1, x_2; y_1, y_2	coordinates of starting and ending points of displacement discontinuity
x_s, y_s	coordinates of the source point of the line force
z, z_s	complex variable to define a field point and source point of the line force
z_1, z_2	complex variables to define starting and ending points of displacement discontinuity

Greek

α_j, β_j, γ_j	constants related to the profile of displacement discontinuity
Γ_1, Γ_2	complex functions
Γ_{1p}, Γ_{2p}	complex functions corresponding to particular solution
Γ_{1c}, Γ_{2c}	complex functions corresponding to complementary solution
ε_{xx}, ε_{yy}, γ_{xy}	strain components
μ	shear modulus
ν	Poisson's ratio
σ^k_{pq}	pq component of stress induced by a line force in k-direction
$\sigma^k_{pq,j}$	derivative of pq-component of stress induced by a line force in k-direction with respect to coordinate j
σ_{xx}, σ_{yy}, σ_{xy}	stress components
$\phi()$, $\psi()$	complex functions
$\phi_p()$, $\psi_p()$	complex functions which describe the particular solution in an infinite plane
$\phi_c()$, $\psi_c()$	complex functions which describe the complementary solution of the half plane
Ω_1, Ω_2	complex functions
Ω_{1p}, Ω_{2p}	complex functions corresponding to particular solution
Ω_{1c}, Ω_{2c}	complex functions corresponding to complementary solution

Acronyms

2D	two-dimensional
3D	three-dimensional
BEM	boundary element method
DD	displacement discontinuity
FEM	finite element method

crack propagation [11]. Moreover, the growth of short fatigue cracks has been studied by 2D DD BEM [12]. Axisymmetric crack problems have been analyzed with the axisymmetric DD method [13].

DD-based BEM analyses have been applied to a variety of problems in geology, especially those involving faults. These models have been used to simulate the behavior and interaction of intersecting faults in both 2D and 3D [14,15], and the