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Re-derivation of plasticity interaction for combined loading under significant levels of elastic follow-up

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ABSTRACT

It is a common need to assess crack-like defects under combined primary and secondary loads. A range of simplified methods exist to describe this interaction. Differences between these methods can be related to how elastic follow-up was treated or modelled. The work presented here compares finite element analyses under large levels of elastic follow-up to assessment methods developed for use in the R6 defect assessment procedure. It is shown that the approaches by James et al., Song et al. and Ainsworth were not always accurate for the extreme conditions considered. However, an improved fit was found from a re-derivation of the Ainsworth approach.

1. Introduction

To assess the criticality of a crack-like defect in a component, it is necessary to consider the value of crack driving force, which can then be compared to a representative material's fracture toughness. The value of the crack driving force used can be determined from elastic–plastic finite element analysis or, more conveniently, by making use of simplified methods such as those contained in the R6 assessment procedure [1]. These assessments must take into account all applied loads, including the combined effect of primary and secondary loads acting to open the crack. For pressurised plant components, primary loads arise from applied forces such as internal pressure; secondary loads typically arise from weld residual stresses and thermal stresses.

A range of methods are currently available within R6 [1] to describe how these primary and secondary stresses combine, which include methods described by the multiplicative V parameter, with further approaches suggested for the next revision. These new approaches include the methodology detailed by Ainsworth [2], Song et al. [3] and James et al. [4]. Each of these methods, including the existing R6 methods, has different levels of associated conservatism. The main reason for these different levels of conservatism is the underlying theory, or fit to finite element analyses, that have been used to define each respective approach. This is because the effect of elastic follow-up may be significantly different over a range of cases (i.e. geometry, loading and material), which will lead to different levels of plastic enhancement to the contribution of the secondary stress to the total crack driving force. Indeed, the existing V-Factor approach within R6 [1] to detail the influence of primary and secondary stresses on fracture is generally considered conservative. However, even for the generally conservative [5] R6 V-Factor approach there have been cases that have shown non-conservatism with excessive levels of elastic follow-up (e.g. [6]). This has led the guidance in R6 [1] for such cases to suggest treatment of secondary stresses with a large

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Nomenclature

Latin symbols

a	crack depth
E	Young's modulus
$f(L_r)$	function describing the R6 failure assessment curve based on L_r
$f(\beta)$	function describing the R6 failure assessment curve based on β
J	energy release per unit area crack growth under elastic plastic conditions
J^p	value of J defined from primary loads alone
K_I^p	elastic stress intensity factor for primary loads alone
K_I^s	elastic stress intensity factor for secondary loads alone
K_{mat}	materials fracture toughness
K_r	effective elastic stress intensity factor normalised by K_{mat}, assessment location on R6 FAD
K_J	effective total elastic–plastic stress intensity factor
K_J^p	elastic–plastic stress intensity factor for primary loads alone
K_J^s	elastic–plastic stress intensity factor for secondary loads alone
L_r	measure of primary load (P/P_L), assessment location on R6 FAD
L_r^{max}	maximum value of L_r allowed before
n	strain hardening index of Ramberg Osgood stress–strain relation
P	applied primary load
P_L	plastic limit load of the cracked structure
R	pipe radius
t	wall thickness
T	applied temperature variation
V	R6 plasticity interaction term for combined loading
V_g	plasticity interaction term for combined loading defined by James [5]
V_0	ratio of K_J^s to K_I^s
x	distance through wall thickness
Z	elastic follow-up factor

Greek symbols

β	measure of secondary stress ($\approx \sigma_{ref}^s/\sigma_y$)
β_{max}	value of β corresponding to maximum value of V_0
ε_{ref}^p	primary reference strain
ξ	R6 term to defined V for combined loading
σ_{ref}^p	primary reference stress
σ_{ref}^s	secondary reference stress
σ_y	yield stress
ν	Poisson's ratio

Abbreviations

FAD	Failure Assessment Diagram
FE, FEA	Finite Element Analysis

elastic follow-up as an additional primary stress. This, however, can be prohibitively conservative and an improved estimate of fracture under conditions of significant elastic follow-up would be beneficial.

Elastic follow-up can be considered to occur in cases where the secondary load acting over a sufficiently large length scale such that localised relaxation (e.g. in the vicinity of a crack) does not diminish the influence of the remote stresses; this therefore reduces the level to which the secondary stress localised to the crack tip can be redistributed. In fracture this means that the secondary stress can act more like a primary stress than a secondary stress. It is assumed that this effect of elastic follow-up can be described by a single parameter, Z [2,7]. Qualitatively, the effect of elastic follow-up can be considered by its effect upon a unit cell (such as a finite element) of a simply loaded material. After elastic loading the value of z determines the final elastic–plastic stress and strain state, as shown in Fig. 1. Under pure displacement control conditions the strain will not change but the stress will drop from point (A) to the stress–strain curve and the elastic follow-up is given by $Z = 1$. For the case where the stress behaves as if under load controlled conditions the stress will not change and the strain will continue until the materials stress strain curve is met; this indicates a large elastic follow-up, $Z \gg 1$. In most cases for a secondary stress, however, the actual value of Z will be somewhere between pure load controlled and pure displacement controlled conditions (i.e. $Z > 1$).