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Stress concentration near stiff inclusions: Validation of rigid inclusion model and boundary layers by means of photoelasticity

D. Misseroni, F. Dal Corso, S. Shahzad, D. Bigoni *

University of Trento, via Mesiano 77, I-38123 Trento, Italy

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ABSTRACT

Photoelasticity is employed to investigate the stress state near stiff rectangular and rhombohedral inclusions embedded in a 'soft' elastic plate. Results show that the singular stress field predicted by the linear elastic solution for the rigid inclusion model can be generated in reality, with great accuracy, within a material. In particular, experiments: (i.) agree with the fact that the singularity is lower for obtuse than for acute inclusion angles; (ii.) show that the singularity is stronger in Mode II than in Mode I (differently from a notch); (iii.) validate the model of rigid quadrilateral inclusion; and (iv.) for thin inclusions, show the presence of boundary layers deeply influencing the stress field, so that the limit case of rigid line inclusion is obtained in strong dependence on the inclusion's shape. The introduced experimental methodology opens the possibility of enhancing the design of thin reinforcements and of analyzing complex situations involving interaction between inclusions and defects.

1. Introduction

Experimental stress analysis near a crack or a void has been the subject of an intense research effort (see for instance Lim and Ravi-Chandar [19,20], Schubnel et al. [25], Templeton et al. [27]), but the stress field near a rigid inclusion embedded in an elastic matrix, a fundamental problem in the design of composites, has surprisingly been left almost unexplored (Theocaris [28]; Theocaris and Paipetis [29,30], Reedy and Guess [22]) and has *never* been investigated via photoelasticity.[1]

Though the analytical determination of elastic fields around inclusions is a problem in principle solvable with existing methodologies (Movchan and Movchan [14], Muskhelishvili [15], Savin [24]) detailed treatments are not available and

* Corresponding author. Tel.: +39 0461 882507; fax: +39 0461 882599.
E-mail addresses: diego.misseroni@ing.unitn.it (D. Misseroni), francesco.dalcorso@ing.unitn.it (F. Dal Corso), summer.shahzad@unitn.it (S. Shahzad), bigoni@ing.unitn.it (D. Bigoni).
URL: http://www.ing.unitn.it/~bigoni/ (D. Bigoni).

[1] Gdoutos [11] reports plots of the fields that would result from photoelastic investigation of rigid cusp inclusions, but does not report any experiment,while Theocaris and Paipetis [30] show only one photo of very low quality for a rigid line inclusion. Noselli et al. [16] (see also Bigoni [2], Dal Corso et al. [6]) only treat the case of a thin line-inclusion. Theocaris and Paipetis [29,30] use the method of caustics (see also Rosakis and Zehnder [23]). This method, suited for determining the stress intensity factor, suffers from the drawback that near the boundary of a stiff inclusion the state of strain can be closer to plane strain than to plane stress, a feature affecting the shape of the caustics.

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Nomenclature

Notation

u_i	displacement vector
$u_{i,j}$	denotes the derivative of u_i with respect to the spatial coordinate x_j
ε_{ij}	second-order deformation tensor
σ_{ij}	second-order stress tensor
σ_{xx}^{∞}	far-field uniaxial tensile stress
E	Young's modulus
μ	shear modulus
ν	Poisson's ratio
κ	parameter depending on Poisson's ratio, defining plane stress or plane strain
l_x	dimension of the inclusion along the horizontal axis in Cartesian coordinates
l_y	dimension of the inclusion along the vertical axis in Cartesian coordinates
r, ϑ	polar coordinates centered at the wedge corner
$F(r, \vartheta)$	Airy stress function
A_i	constants for the Airy stress function
γ	power of r in the Airy function for the stress and strain fields
α	semi-angle in the matrix enclosing the wedge
i	imaginary unit
z	complex variable in the physical z-plane
ζ	complex variable in the conformal ζ-plane
$\varphi(z), \psi(z)$	complex potentials
$\varphi^p(z), \psi^p(z)$	perturbed complex potentials
$\omega(\zeta)$	conformal mapping function
$(\cdot)'$	first-order derivative
$(\cdot)''$	second-order derivative
$\overline{(\cdot)}$	conjugate
n	number of vertices of the polygon
R	scaling of the inclusion in the ζ-plane
k_0	translation of the inclusion in the ζ-plane
α_0	rotation of the inclusion in the ζ-plane
k_j	pre-image of the j—th vertex in the ζ-plane
α_j	fraction of π of the j—th interior angle in the ζ-plane
a_j	complex constants of the perturbed complex potential $\varphi^{(p)}(z)$
d_j	complex constants of the conformal map function $\omega(\zeta)$
η	parameter function of the rectangle aspect ratio
b_j, c_j	complex constants of the perturbed complex potential $\psi^{(p)}(z)$
N	fringe number
f_σ	material fringe constant
σ_I	maximum in-plane principal stress
σ_{II}	minimum in-plane principal stress
$\Delta\sigma$	in-plane principal stress difference

the existing solutions [2] lack mechanical interpretation, in the sense that it is not known if these predict stress fields observable in reality[3]. Moreover, from experimental point of view, questions arise whether the bonding between inclusion and matrix can be realized and can resist loading without detachment (which would introduce a crack) and if self-stresses can be reduced to negligible values. In this article we (i.) re-derive asymptotic and full-field solutions for rectangular and rhombohedral rigid inclusions (Section 2) and (ii.) compare these with photoelastic experiments (Section 3).

Photoelastic fringes obtained with a white circular polariscope are shown in Fig. 1 and indicate that the linear elastic solutions provide an excellent description of the elastic fields generated by inclusions up to a distance so close to the edges of the inclusions that fringes result unreadable (even with the aid of an optical microscope). By comparison of the photos shown in Fig. 1 with Fig. 1 of Noselli et al. [16], it can be noted that the stress fields correctly tend to those relative to a rigid line

[2] Evan-Iwanowski [9]) treated the case of a triangular rigid inclusion, Chang and Conway [3] addressed rectangular rigid inclusions, while Panasyuk et al. [21] considered the problem of the stress distribution in the neighborhood of a cuspidal point of a rigid inclusion embedded in a matrix. Ishikawa and Kohno [12] and Kohno and Ishikawa [13] developed a method for the calculation of the stress singularity orders and the stress intensities at a singular point in an polygonal inclusion.

[3] The experimental methodology introduced in the present article for rigid inclusions can be of interest for the experimental investigation of the interaction between inclusions and defects, such as for instance cracks or shear bands, for which analytical solutions are already available (see Piccolroaz et al. [17,18], Valentini et al. [31], Dal Corso and Bigoni [4,5]).