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A novel hybrid approach for level set characterization and tracking of non-planar 3D cracks in the extended finite element method

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ABSTRACT

In this paper, the limitations associated with implicit and explicit representations of cracks in the extended finite element method (XFEM) is recapitulated via numerical explorations followed by the development of a novel hybrid approach for the characterization of non-planar 3D cracks along with its capability demonstration. In the XFEM, the crack geometry is independent of the structural mesh, and is often described implicitly by means of two level set functions. The implicit representation is very convenient for purposes of computing the crack front velocity and for handling situations where the crack front is concave and the velocity vectors may cross. The main difficulty of this implicit description is the formulation of an efficient and robust update scheme for the level set values after a propagation step. On the other hand, the crack geometry can be described by an explicit triangulated mesh which can be easily updated after a propagation step. The explicit representation has its own shortcomings, e.g., difficulties in handling crack overlaps and extraction of crack local coordinates. Given the difficulties associated with the use of either implicit or explicit method for the geometric description of complex crack geometry, a novel hybrid method is developed by a combination of an implicit level set representation of the crack and an explicit triangulated mesh representation. In the hybrid approach, the implicit representation is updated after each propagation step and disconnected crack surfaces are removed using a paint-fill algorithm based on the current explicit representation of the crack. Then, an updated explicit representation is constructed based on the updated implicit representation using the marching cubes algorithm. Finally, the implicit representation is rebuilt from the explicit representation. The use of the explicit representation ensures that the data in the level set representation is generated from a consistent crack description. The effectiveness of the developed hybrid approach is demonstrated by analyzing several 3D crack propagation problems including a quarter-circular crack in a complex helicopter component, a U-shaped crack and an inclined elliptical crack in cuboids, and an inclined edge crack in a three-point bending beam.

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Nomenclature

b_i	jump function parameters
c_{ji}	branch function parameters
H	jump function
K	stress intensity factor
N_i	finite-element shape functions
$\mathbf{p}_i$	grid nodes in the implicit representation
$\mathbf{q}_i$	nearest point on the crack front to the grid node of interest
r	distance from the crack tip/front
$\mathbf{t}_i$	tangent vectors to the crack front
U_i	nodal displacement parameters
$\mathbf{v}_i$	velocities of the sampling point along the crack front
$\mathbf{x}_i$	sampling point along the crack front
δ	crack tip opening
θ	angle to the tangent plane at the tip/front
μ	shear modulus
φ	normal level set function
ψ	tangential level set function
Ψ_j	branch functions

1. Introduction

The extended finite element method (XFEM) [1,2] has been extensively used to introduce jumps, kinks, singularities, and other nonsmooth features within finite element models independently of the mesh [3]. It is achieved by means of special functions enriching the standard finite element (FE) shape functions. The XFEM has been more and more commonly used for applications in fracture mechanics [4–11]. The XFEM belongs to the category of the continuum-based approach for analysis of solid mechanics problems while the other category, the discontinuous approach, also includes very powerful methods for fracture mechanics simulations, e.g., the peridynamics method [12,13], and the volume-compensated particle method [14,15]. Using the discontinuous approach, crack initiation and propagation processes can be modeled in a unified way based on bond breaking and removal rules and they avoid singularity-related issues. On the other hand, the main difficulties of the continuum-based fracture mechanics approach are the stress singularity at the crack front and modeling discontinuities in the displacement field, which requires expensive remeshing in the conventional finite element methods.

Presence of a crack means the discontinuity in the displacement field across the crack surface. Through a local enrichment of the FE shape functions of the elements surrounding the crack, the XFEM introduces this displacement discontinuity and also the singularity of the stress field at the crack front. The locations of the crack surface and front are often defined implicitly in the XFEM by means of the level set method [16–18]. Using the level set method an interface is represented as the zero level set of some higher dimensional function. In the XFEM, two level set functions are used to describe the crack (Fig. 1): (1) the normal level set function, $\varphi(\mathbf{x}, t)$, which gives the signed normal distance between point $\mathbf{x}$ and the crack surface, and (2) the tangential level set function, $\psi(\mathbf{x}, t)$, which gives the signed normal distance between point $\mathbf{x}$ and the surface obtained

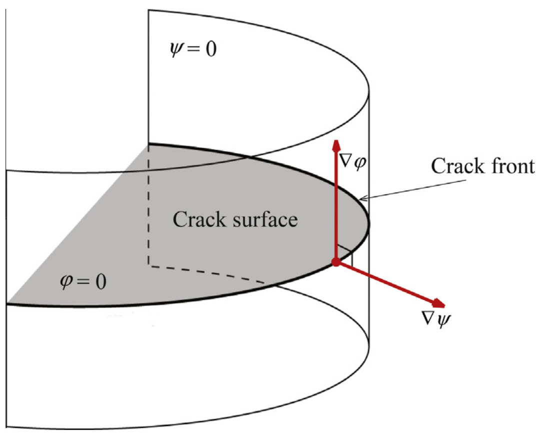

Fig. 1. The implicit representation of a crack in the XFEM using two level set functions.