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Crack analyses in porous piezoelectric brittle materials by the SBFEM

Jan Sladek [a,*], Vladimir Sladek [a,1], Slavomir Krahulec [a,1], Chongmin Song [b]

[a] Institute of Construction and Architecture, Slovak Academy of Sciences, 84503 Bratislava, Slovakia
[b] School of Civil and Environmental Engineering, The University of New South Wales, Sydney, NSW 2031, Australia

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ABSTRACT

The scaled boundary-finite element method (SBFEM) is employed to analyze cracks in porous piezoelectric solids. A large (magistral) crack and a short crack emanating from a single pore are analyzed. These cracks with their tips in the solid skeleton have the weakening effects on the fracture strength. The crack tip region is considered as a subdomain where both the crack tip and pores are modeled inside the piezoelectric skeleton. The remaining part of the analyzed domain is modeled with effective material properties obtained from the analysis on the representative volume element (RVE). A regular distribution of circular voids is considered in the numerical analyses. The scaled boundary-finite element method (SBFEM) is applied to solve all the boundary value problems.

1. Introduction

Defects, cracks or voids commonly occur in piezoelectric ceramics. The presence of voids has an influence on material properties and functionality of piezoelectric structures. In this regard, there have been numerous experimental studies on the properties of porous piezoelectric materials and, in particular, the influence of porosity on them [1–6]. While there exist a large number of experimental results, there are only a few computational models for studying these materials. The finite element method (FEM) has been applied by Kar-Gupta and Venkatesh [7] to study the effect of porosity on the electromechanical response of piezoelectric materials. The commercial FEM computer code, ANSYS, was utilized by Li et al. [8] to study the effective properties of porous piezoelectric materials on a cubic unit cell model. The boundary node method (BNM) was also successfully applied to porous piezoelectric materials to evaluate the effective material properties [9].

The fracture of porous materials is a difficult problem and it is quite seldom analyzed in the literature. Contradictory results are often reported in the fracture mechanics of porous materials with either strengthening or weakening effects [10–13]. If the pores are small compared to the structure and the crack length, a weakening effect of the effective elastic parameters is observed with increasing volume fraction of pores. On the other hand, for a short crack with each end at the edge of a circular cavity, there is apparent toughness enhancement due to blunting of the circular hole. Krstic [14] analyzed short cracks emanating either from a single pore or a cluster of pores. The concept of pores being responsible for the

* Corresponding author. Tel./fax: +421 54773548.
E-mail address: jan.sladek@savba.sk (J. Sladek).
[1] Tel./fax: +421 54773548.

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Nomenclature

a, b width and height of RVE
a, W, H crack-length, width and height of strip
c crack-length emanated from circular void
c_{ijkl}, c_{ijkl}^{eff} elastic material tensor, effective elastic material tensor
$\{c_i\}, \{c_s\}$ integration constants, singular part of integration constants
e_{jkl}, e_{jkl}^{eff} piezoelectric material tensor, effective piezoelectric material tensor
f volume fraction of voids
$\{\overline{q}(\xi)\}$ generalized force
h_{jk}, h_{jk}^{eff} dielectric material tensor, effective dielectric material tensor
n number of nodes in SBFEM, number of voids in numerical results
n_i normal component
r, θ polar coordinates
r_0 radius of the circular void
s length of side of square area around crack tip
t_i traction
$\tilde{t}$ natural boundary condition for mechanical field
u_i, $\{\overline{u}\}$ displacements, generalized displacements
$\tilde{u}_i$ essential boundary condition for mechanical field
x_i global cartesian coordinates
x, y cartesian coordinates of nodal points on boundary element
$[C]$ matrix of material coefficients
D_i electric displacement
D_0 electrical load
E_j electric field vector
$[E^0]$, $[E^1]$, $[E^2]$ coefficient matrices
$[I]$ identity matrix
$[\hat{J}(\xi,\eta)]$, $[J(\eta)]$ Jacobian matrix
$[K]$ stiffness matrix
$\{K(\theta)\}(K_I(\theta), K_{II}(\theta), K_{IV}(\theta))$ stress and electric displacement intensity factors
K_{In}, K_{IIn}, K_{IVn} normalized stress and electric displacement intensity factors
$L(\theta)$ radial coordinate of boundary point with angular coordinate θ
$[L]$ linear differential operator
$[N(\eta)]$ matrix of shape functions
O scaling center
R volume density of free charges
S, S^e boundary, boundary line element
V, V^e analyzed domain, triangular sector
W electric enthalpy density
X_i body force vector
$[Z]$ Hamiltonian coefficient matrix
ε_{ij}, $\{\overline{\varepsilon}\}$ strain tensor, generalized strain
λ_i (λ_n, λ_p, λ_s) eigenvalues (negative, positive, singular)
ξ, η scaled boundary coordinates (dimensionless radial coordinate, local (circumferential) coordinate)
ρ mass density
σ_{ij}, $\{\overline{\sigma}\}$ stress tensor, generalized stress
σ_0 mechanical load
ϕ electric potential
$\tilde{\phi}$ essential boundary condition for electrical field
Γ (Γ_u, Γ_t, Γ_p, Γ_q) global boundary (part of global boundary with prescribed displacements, traction vector, electric potential, normal component of electric displacement vector)
$[\Psi]$ ($[\Psi_i^u]$, $[\Psi_s^u]$, $[\Psi_i^q]$) transformation matrix (displacement and electric potential modes; singular part of displacement and electric potential modes, modes of generalized forces)

initiation of failure is extensively studied there. In this case, the crack size is small compared to the pore/hole radius. Both crack problems with a large (magistral) and short crack in a voided piezoelectric medium are analyzed in this paper. The crack tips, however, are not at the edges of circular voids. It is evident that elastic and piezoelectric properties are weakened