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Comparison of two numerical procedures for solution of the integro-differential equation of flat crack problem

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Igor Orynyak [a,1], Anatolii Batura [a,*], Andrii Oryniak [b,2]

[a] *G.S. Pisarenko Institute for Problem of Strength, NAS of Ukraine, 01014, 2 Timiryazevs'ka Str, Kyiv, Ukraine*
[b] *National Technical University of Ukraine "KPI", 03056, 37 Prosp. Peremohy, Kyiv, Ukraine*

A R T I C L E   I N F O

A B S T R A C T

General approach for numerical treatment of the integro-differential equation of flat crack problem is considered. It consists in presenting the crack surface loading as a set of polynomial functions of two Cartesian coordinates while the corresponding crack surface displacements are chosen as similar polynomials multiplied by the function of form (FoF) to reflect the required singularity of their behavior. Two methods of getting the relations matrixes between these two sets are examined: the first is classical one (when initially the Laplace operator is analytically applied to the integral part of equation and later the resulting hypersingular equation is considered); and the second one is so called direct method (values of the integral are calculated at chosen points of the crack surface, and then these are fitted by polynomial series with subsequent application of the Laplace operator to them).

In both these methods the most efforts are devoted to the choice of the FoFs. Three different types of them are tested as to accuracy of results. The first is a usual one when the conditional center of crack is chosen and the FoF is taken as a square root of 1 minus squared relative polar radius of considered surface point. What is unusual here is investigation of shift of the center even for the circular crack. The second one is presentation of the FoF as a square root of products of equations of straight and circular lines of crack boundary. And the third one is based on new idea to use the Burns–Oore FoF, previously suggested in their famous 3D weight function method.

Comprehensive investigation of the accuracy of above methods with different combination of FoFs on examples of circular, elliptic, semicircular and square cracks are performed.

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1. Introduction

1.1. The problem statement

Mode I flat crack problem can be reduced to the solution of the following integro-differential equation [1,2]:

$$-p(x,y)\frac{4\pi(1-\mu^2)}{E} = I(x,y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\iint_{(S)} \frac{U(\xi,\eta)d\xi d\eta}{R^*(x,\xi,y,\eta)} \tag{1}$$

* Corresponding author. Tel.: +380 50 387 92 92; fax: +380 44 284 94 81.
   E-mail addresses: igor_orinyak@yahoo.com (I. Orynyak), asbatura88@gmail.com (A. Batura), oryand@ipp-centre.com (A. Oryniak).
[1] Tel.: +380 50 163 69 68; fax: +380 44 284 94 81.
[2] Tel.: +380 50 935 00 05.

Nomenclature

a_v	coefficients of interpolation function on the integration line
A_{ij}	coefficients of polynomial presentation of the crack surface displacement
B_{nm}	coefficients of the polynomial presentation of crack surface loading
E	module of elasticity
$f(\rho)$	interpolation function on the integration line
F_I	dimensionless SIF value
$J = max(i + j = n + m)$	the maximum degree of stress and displacement polynomials in the hypersingular method
$J_{integral} = J + 2$	maximum degree of interpolation polynomial for values obtained by integrating in the direct method
K_I	SIF value
(M_1)	direct solution matrix, which gives stresses for the chosen set of displacement
$(M_2) = (M_1)^{-1}$	inverse solution matrix – the main goal of the work, it provides displacements for the chosen stresses
O	conditional center of the crack
$O^q(x_q, y_q)$	collocation point (the value of integral is numerically calculated in it)
$p(x, y)$	field of normal stress acted on the crack surface
r	radial coordinate of considered point of the crack surface with respect to the conditional center O
r_{loc}	radial coordinate in local system with respect to the collocation point $O^q(x_q, y_q)$
R	radial coordinate of the crack contour point related to the considered point of crack surface, r, (the crack center O, considered surface point and crack contour point are on the same line)
S	area of the flat crack
$U(\xi, \eta)$	field of the crack opening displacements
V	the maximum degree of interpolation polynomial on the line of integration
$(x, y), (\xi, \eta)$	sets of Cartesian coordinates with origin in the point O
Δ	the shortest distance between the crack front and crack surface point considered
μ	Poisson's ratio
ρ	normalized local radial coordinate of arbitrary point on the integration line, linearly dependent on r_{loc}
ρ_0	normalized local radial coordinate of the collocation point
l	semi-length of the line of integration
$\Omega, \Omega_1, \Omega_2, \Omega_3$	different types of the FoF of crack opening displacement

here S is an area of the flat crack of arbitrary shape in the infinite isotropic body; (x, y) and (ξ, η) are two sets of Cartesian coordinates originated in point O (Fig. 1); $p(x, y)$ is a normal stress (loading) acted on the crack surface and $U(\xi, \eta)$ is a displacement of crack surface points; R^* is a distance between the point of load application and considered point of crack surface, $R^* = \sqrt{(x - \xi)^2 + (y - \eta)^2}$, E is a module of elasticity, μ is a Poisson's ratio.

Given that crack opening displacement (COD) field $U(\xi, \eta)$ is available in analytical form, the SIF value in the arbitrary contour point with angular coordinate θ can be found from well-known limit transition:

$$K_I(\theta) = \frac{\sqrt{\pi}}{2\sqrt{2}} \cdot \frac{E}{1 - \mu^2} \lim_{\Delta \to 0} \frac{U(r, \varphi)}{\sqrt{\Delta}} \tag{2}$$

where Δ is the shortest distance between the crack front point with coordinate $\varphi = \theta$ and considered crack surface point.

In spite of relative simplicity of Eq. (1), to obtain the universal solution for cracks of arbitrary shape is extremely complex task. Up to day, effective fundamental analytical solution for the COD was obtained only for the circular crack, while for the crack of elliptical configuration the solutions were obtained only for polynomial loading. With respect to the crack of complex form the available general numerical schemes of treatment of (1) are scarce and cannot be considered as the universal ones.

Now, the intensive development of computer capacities makes it possible to elaborate comprehensive approaches to the solution of Eq. (1) for the arbitrary crack shape. At the same time, great number of universal commercial FEM based softwares are evolved, which are able to achieve the very high level of accuracy. So the following question is quite natural: "Are theoretical methods (e.g. bases on solution of (1)) really important and demanded now?"

From the author's point of view, theoretical methods are still of a high demand and can be used in the following:

– As the reference results for numerical methods to test the accuracy and adequacy of SIF calculations. Such task is quite demanded, because even generally precise numerical method may give inadequate results with respect to the means of local meshing at the crack tip and their relation with the global geometry model.