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The roles of cohesive strength and toughness for crack growth in visco-elastic and creeping materials

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ABSTRACT

A cohesive-zone analysis for crack propagation in a linear visco-elastic/creeping material is presented. The concept of a viscous fracture length is defined; this serves an analogous role to the elastic fracture length in determining the conditions under which fracture is controlled by the continuum crack-tip stress field. It is shown that there are two regimes for viscous crack growth. The first regime occurs in the limit of small viscous fracture lengths, when the crack-tip stress field has a region exhibiting the inverse square-root dependence expected from classical linear fracture mechanics. In this regime, the crack velocity is proportional to the fourth power of the stress-intensity factor. This is consistent with an existing analytical model developed for crack growth in linear polymers. The second regime occurs for large viscous fracture lengths, where classical fracture mechanics is not appropriate. In this regime, the crack velocity has a weaker dependence on the applied load, and can be modeled accurately by the solution to the problem of a viscous beam on an elastic foundation. At higher crack velocities, when the viscous fracture length exceeds the elastic fracture length, the expected transition to elastic fracture occurs.

1. Introduction

Modelling crack growth requires an understanding of which parameters control fracture. For example, linear-elastic fracture mechanics (LEFM) is a continuum model in which crack growth is controlled by an energy criterion; the fracture load depends only on the modulus, E, the toughness, Γ, and a dimension describing the physical size of the geometry, h (in addition to a non-dimensional description of the geometry). Generally speaking, this approach works when there is any region near the crack tip where the stresses can be described reasonably well by the continuum singular field. However, more generally, analysis of fracture requires the introduction of an additional parameter. This additional parameter can often be expressed in terms of a length associated with the fracture process. Sometimes, this length may enter the problem directly as a length over which the continuum approach breaks down, or as a critical crack-tip displacement for crack propagation. In cohesive-zone models of fracture, it enters in a dimensional fashion through a cohesive strength $\hat{\sigma}$, giving an elastic fracture length defined by $E\Gamma/\hat{\sigma}^2$, which has a unit of length.

The original motivation for this study was to determine the fracture parameters that control crack growth in a creeping solid. We addressed this by conducting a cohesive-zone analysis with a linear visco-elastic material. This analysis shows that, in contrast to when fracture is controlled by elasticity, there are no conditions under which crack growth can be modeled in a

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Nomenclature

E	Young's modulus
Γ	toughness of interface
h	thickness of beams
$\hat{\sigma}$	cohesive strength of interface
M_∞	applied moment
T_n	normal traction
k	spring constant
v	displacement
$\tilde{v}$	non-dimensional displacement, v/h
z	distance ahead of crack tip
$\tilde{z}$	non-dimensional distance ahead of crack tip
β_o	$(3kh/E)^{1/4}$
ζ	elastic fracture length
$\tilde{\zeta}$	elastic fracture length scale, ζ/h
η	uniaxial viscosity
t	time
$\dot{a}$	crack velocity
λ_o	$\left(12kh^2/\eta\dot{a}\right)^{1/5}$
ζ_v	viscous fracture-length scale
$\tilde{\zeta}_v$	non-dimensional viscous fracture-length scale
$\dot{\varepsilon}_{ij}$	strain-rate tensor
ν	Poisson's ratio
σ'_{ij}	deviatoric stress tensor
C^*	K^2/η where K is the stress-intensity factor

viscous or creeping material without introducing a fracture length of some description. This is true even in regimes where the crack-tip stresses exhibit a region that can be described by the continuum singular field. This conclusion is consistent with the observations of Rice [1], and with the comments of McCartney [2,3] in response to the work of Christensen [4–6].

Time-dependent crack growth has historically been studied in two distinct areas of research: creep rupture of metals and ceramics, and fracture of polymers. Very different frameworks have been developed in each of these two areas to describe what is essentially the same problem of time-dependent crack growth. The different perspectives provided by the frameworks have resulted in what might appear to be contradictory conclusions about whether time-dependence is a desirable attribute from a fracture perspective or not. The creep-rupture literature tends to describe the problem in terms of how the time-dependent properties of a material result in sub-critical cracking at low driving forces (an apparent weakening). Conversely, the polymers literature often tends to describe the problem in terms of how the time-dependent properties of a material result in an increased rate of energy dissipation (an apparent toughening). This is, of course, merely a manifestation of the classic question of whether one is more interested in the toughness or the strength of a material system.

Crack-growth models for creeping materials are often formulated in terms of the nucleation and growth of damage in the form of cavities ahead of a crack tip [7–9]. If it is assumed that the damage is embedded within a crack-tip stress field appropriate for a creeping solid, its growth can be linked to the deformation of the surrounding material [10–13]. In particular, crack advance occurs when the crack-tip region has deformed sufficiently to accommodate a critical level of damage, which may, or may not, be time-dependent. The associated analyses always require the introduction of a characteristic length beyond any continuum description of the geometry, to ensure dimensional consistency. For example, in the model of Cocks and Ashby [10], this length scale is the distance over which the damage is assumed to grow under the influence of the crack-tip stress field.

The results of models for the crack velocity in creeping materials depend on the underlying assumptions about how the damage interacts with the stress field, and how the stresses evolve at the crack tip. However, the different models share a common aspect in that the crack velocity depends on the crack-tip loading parameter for creep, C^*, which is the time-dependent analog of the J-integral [14,15]. The effect of creep/viscosity is to cause sub-critical crack growth until the cracks are long enough for the elastic-fracture criterion to be met, when catastrophic failure can occur. The implication of this perspective is that viscosity weakens a material, since it provides a mechanism to accommodate the growth of damage to a critical value at relatively low loads.

The mechanics of time-dependent fracture of polymers is essentially identical to that of creep rupture. However, much of the literature often focuses on the concept of a rate-dependent toughness [16–18], rather than on how the crack velocity varies with loading parameter. The viscous energy dissipated at the crack tip is seen as contributing to the toughness, and the size of the crack-tip viscous zone depends on the crack velocity [19]. The implication of this perspective is that viscosity toughens a material, since it provides a mechanism to dissipate additional energy at the crack tip.