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Technical Note

Solution for a crack embedded in thermal dissimilar elliptic inclusion

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ABSTRACT

This paper provides a solution for a crack embedded in the thermal dissimilar elliptic inclusion. The whole medium is composed of a cracked inclusion and an infinite matrix. The inclusion and the matrix have different elastic properties and temperature distribution. The embedded crack is replaced by a very slender elliptic contour. The complex variable and the conformal mapping are used in the paper. From the evaluated stress concentration factor at the crown point, we can get the stress intensity factor at the crack tip from an existing equation.

1. Introduction

Many researchers studied the inclusion problem or inhomogeneity in plane elasticity [1–5]. A few of the studies were devoted to a crack embedded in the inclusion.

A solution for crack in a confocal elliptic inhomogeneity embedded in an infinite medium was proposed [1]. In the solution, the conformal mapping method is used. Therefore, the complex potentials defined on the ring region of mapping plane can be expanded into a Laurent series. From the traction free condition along the crack, the continuity conditions along interface and the remote loading condition, the boundary value problem is solved accordingly. After using the complex variable method and the conformal mapping, the uniform stress state inside an inclusion of arbitrary shape in a three-phase composite was studied [2].

A solution in plane elasticity for multiple elliptic layers with different elastic properties was suggested [3]. The complex variable method and the conformal mapping are used. The continuation conditions for traction and displacement along the interface are satisfied in a weaker form. Many numerical results are presented. A solution for thermal elliptic inclusion in plane elasticity was studied [4]. There is a difference for the temperatures in the inclusion and the matrix. Two types of temperature distributions in the inclusion, namely the constant distribution and the linear distribution, are assumed. The problem is solved in a closed form. A comprehensive review for recent works on inclusions was provided [5]. The review concludes with an outlook on future research directions.

This paper provides a solution for a crack embedded in thermal dissimilar elliptic inclusion. The whole medium is composed of a cracked inclusion and an infinite matrix. The inclusion and the matrix have different elastic properties and temperature distribution. The embedded crack is replaced by a very slender elliptic contour. The complex variable and the conformal mapping are used in the paper. The complex potentials on the ring region of the mapping plane are expanded into

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a Laurent series. The boundary condition and the continuity conditions for the traction and the displacement are proposed, and they are satisfied in a weaker form. The concentrated stress at the crown point of notch can be evaluated accordingly. The stress intensity factor at the crack tip can be evaluated from the stress concentration factor by using an existing equation. The stress intensity factor depends on (a) the geometry of the medium, (b) the elastic constants on different portions of the medium and (c) the temperature distribution on different portions of the medium. Several numerical results are presented.

2. Analysis

A complex potential in the thermal field is introduced as follows [4,6]

$$h(z) = t(x,y) + iq(x,y), \qquad (\text{with } \nabla^2 t(x,y) = 0, \nabla^2 q(x,y) = 0) \tag{1}$$

where $t(x,y)$ is the temperature distribution, and $q(x,y)$ is the thermal stream function. Clearly, if the temperature function $t(x,y)$ is given beforehand, the stream function $q(x,y)$ can be determined by a difference of constant.

From Eq. (1), we can define a complex variable function as follows

$$H(z) = \int_{z_0}^{z} h(z)\,dz = T(x,y) + iQ(x,y), \qquad \text{or} \quad h(z) = H'(z) \tag{2}$$

In thermal plane elasticity, the stresses, resultant forces and displacements can be expressed as [4,6]

$$\sigma_x + \sigma_y = 4Re\phi'_*(z),$$

$$\sigma_y - \sigma_x + 2i\sigma_{xy} = 2\left\{z\overline{\phi''_*(z)} + \overline{\psi'_*(z)}\right\} \tag{3}$$

$$-Y + iX = \phi_*(z) + z\overline{\phi'_*(z)} + \overline{\psi_*(z)} \tag{4}$$

$$2G(u + iv) = \kappa\phi_*(z) - z\overline{\phi'_*(z)} - \overline{\psi_*(z)} + 2G\alpha H(z) \tag{5}$$

where $\phi_*(z)$ and $\psi_*(z)$ denote two complex potentials. In Eq. (5), $H(z)$ is the function defined by Eq. (2), G is the shear modulus of elasticity, and two constants are defined by

$$\begin{aligned} &\kappa = (3-\nu)/(1+\nu), \ \alpha = \alpha_t, \quad (\text{for the plane stress problem}) \\ &\kappa = 3 - 4\nu, \ \alpha = (1+\nu)\alpha_t, \quad (\text{for the plane strain problem}) \end{aligned} \tag{6}$$

In Eq. (6), ν is the Poisson's ratio, and α_t is the thermal expansion coefficient per unit elevation of temperature.

In the analysis, the conformal mapping function $z = \omega(\varsigma)$ is used, for example, which maps the ellipse with its exterior region in the z-plane into the unit circle with its exterior region in the ς-plane (Fig. 1).

In the following analysis, we denote

$$\phi(\varsigma) = \phi_*(z)|_{z=\omega(\varsigma)}, \qquad \psi(\varsigma) = \psi_*(z)|_{z=\omega(\varsigma)} \tag{7}$$

Clearly, after using the mentioned conformal mapping, from Eqs. (1)–(7) we have

$$\sigma_x + \sigma_y = 4Re\frac{\phi'(\varsigma)}{\omega'(\varsigma)}$$

$$\sigma_y - \sigma_x + 2i\sigma_{xy} = 2\left(\frac{\overline{\omega(\varsigma)}(\phi''(\varsigma)\omega'(\varsigma) - \phi'(\varsigma)\omega''(\varsigma))}{(\omega'(\varsigma))^3} + \frac{\psi'(\varsigma)}{\omega'(\varsigma)}\right) \tag{8}$$

$$F = -Y + iX = \phi(\varsigma) + \omega(\varsigma)\overline{\frac{\phi'(\varsigma)}{\omega'(\varsigma)}} + \overline{\psi(\varsigma)} \tag{9}$$

$$2G(u + iv) = \kappa\phi(\varsigma) - \omega(\varsigma)\overline{\frac{\phi'(\varsigma)}{\omega'(\varsigma)}} - \overline{\psi(\varsigma)} + 2\alpha GB(\varsigma), \quad \left(\text{with } B(\varsigma) = H(z)|_{z=\omega(\varsigma)}\right) \tag{10}$$

From Eqs. (8)–(10) we see that, if one obtains the complex potentials $\phi(\varsigma)$ and $\psi(\varsigma)$ in the mapping plane, one can get the stress and displacement in the physical plane.

In the study, the whole region is composed of two phase composites (Fig. 1(a)). The inclusion bounded by an ellipse Σ_2 with two semi-axes $a_2,\ b_2$ contains a crack with length "2c" $\left(c = \sqrt{a_2^2 - b_2^2}\right)$. The crack configuration is denoted by Σ_{1c}. For the cracked inclusion bounded by the crack Σ_{1c} and the elliptic contour Σ_2, we have the following elastic constants and the