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A plastic-damage constitutive model for the finite element analysis of fibre reinforced concrete

Iulia C. Mihai [a,*], Anthony D. Jefferson [a], Paul Lyons [b]

[a] Cardiff University, School of Engineering, Queen's Buildings, The Parade, CF24 3AA, UK
[b] LUSAS, UK

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ABSTRACT

A unique constitutive model for fibre reinforced concrete (FRC) is presented, which combines a number of mechanics-based sub models for the simulation of directional cracking, rough crack contact and the crack-bridging action of fibres. The model also contains a plasticity component to simulate compressive behaviour. The plasticity component employs a frictional hardening/softening function which considers the variation of compressive strength and strain at peak stress with fibre content. Numerical results from a range of single-point and finite element simulations of experimental tests show that the model captures the characteristic behaviour of conventional fibre reinforced concrete with good accuracy.

1. Introduction

The addition of randomly distributed short fibres in a cementitious matrix can significantly improve the fracture properties of these fibre reinforced cementitious composites (FRCC). For the moderate fibre percentages used in many commercial mixes (0.5–2% by volume), the uniaxial tensile and compressive strengths are only increased by a relatively small amount, but it is the overall toughness, energy absorption and crack width control characteristics that can be dramatically improved by the introduction of these fibres. The degree of this enhancement depends mainly on the content and geometry of the fibres as well as upon the fibre-concrete matrix bond properties. This substantial increase in toughness and fracture resistance derives from the debonding and pull-out of fibres from the cementitious matrix. The underlying failure mechanism of cementitious composites, reinforced with randomly oriented discontinuous fibres, is also largely governed by fibre pull-out. This may or may not be accompanied by fibre rupture, depending on the fibre geometry and fibre-cement matrix interface properties [1–3]. When a crack opens, the fibres crossing the crack plane begin to debond and are subsequently pulled out (i.e. the fibres slide relative to the concrete matrix). In this process, they can be considered to apply closure tractions to the crack faces thus stabilising the crack growth. Via these crack-bridging mechanisms, the fibres continue to transfer stresses between the two crack faces until their complete pull-out.

Several models that describe the pull-out of randomly distributed short fibres from a cementitious matrix have been proposed [1,2,4–7]. Naaman et al. [4,5] investigated the influence of fibre–matrix bond properties on the behaviour of FRCCs using an analytical model, since it had become apparent from experimental investigations that the behaviour of FRC,

* Corresponding author.
E-mail address: mihaiic1@cardiff.ac.uk (I.C. Mihai).
URL: http://www.lusas.com (P. Lyons).

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Nomenclature

Symbol	Meaning
a_c, c_{c1}, c_{c2}	constant in hardening plasticity function
a_f, a_{f0}, a_{flim}	aspect ratio of fibres and upper & lower limits of this ratio
$A_r(\theta)$	function in plastic yield function (Appendix B)
b, c	yield function constants (Appendix B)
b_r	biaxial to uniaxial compressive strength ratio
c	shear stress intercept (Appendix A)
c_1	softening curve constant (Appendix A)
c_f, g	constants in the crack-bridging stress function
$c_{v1}, c_{v2}, c_{a1}, c_{v\varepsilon1}, c_{v\varepsilon2}, c_{v\varepsilon3}, c_{a\varepsilon1}$	plasticity parameters (Table 2)
$\tilde{\mathbf{C}}$	elastic crack-band compliance matrix
$\tilde{\mathbf{C}}_{dfs}$	local crack compliance
d_f	diameter of fibres
$\tilde{\mathbf{D}}$	local elastic constitutive matrix
$\boldsymbol{D}_e$	elasticity tensor matrix
$\tilde{\mathbf{e}}_c$	local embedment strain
E, E_f, E_m	Young's modulus of FRC, of fibres and of plain concrete matrix respectively
E_{sf}	effective elastic stiffness of the fibres crossing a crack plane
f_{c0}, f_c	uniaxial compressive strength of plain concrete and of FRC
f_{t0}	uniaxial tensile strength of plain concrete
f_{snub}	snubbing coefficient
$F(\boldsymbol{\sigma}, Z)$	yield function (Appendix B)
G	shear modulus (Appendix A)
$G(\boldsymbol{\sigma}, Z)$	plastic potential (Appendix B)
G_f	fracture energy
h	physical crack-band thickness, which equals the width of the fracture process zone
H_c	contact reduction function
$\boldsymbol{I}$	identity matrix
I_1	1st stress invariant (Appendix B)
J_2	2nd deviatoric stress invariant (Appendix B)
ℓ_{ch}	element characteristic length
L_f	length of fibres
m_g	slope of conical part of contact function
m_{ful}	multiplier on ε_0 which controls the effective end of a shear contact region
$\boldsymbol{N}$	stress transformation matrix
$\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$	crack plane coordinate unit vectors
r_σ	relative shear stress (Appendix A)
$\tilde{u}_{0d}, \tilde{u}_{0p}$	crack-openings at the end of debonding and pull-out stage respectively
$\tilde{\mathbf{u}}$	crack plane displacement vector
$\breve{\mathbf{u}}$	inelastic component of the crack plane displacement vector
V_f	volume fraction of fibres
$\alpha, \gamma, \rho, \rho_c$	yield function constants (Appendix B)
α_p	pull-out reduction coefficient (Eq. 10b)
β_f	fibre–matrix interface parameter. Frictional sliding hardening parameter
ε_0	strain at the effective end of softening curve
ε_{c0}, ε_c	strain at uniaxial peak compression for plain concrete and FRC
$\tilde{\varepsilon}_{0d}, \tilde{\varepsilon}_{0p}$	strains at the end of debonding and pull-out respectively
$\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_e, \boldsymbol{\varepsilon}_p$	Cartesian total strain, elastic strain and plastic strain vectors (6 × 1)
$\tilde{\boldsymbol{\varepsilon}}, \tilde{\boldsymbol{\varepsilon}}_e$ $\breve{\boldsymbol{\varepsilon}}$	crack plane total strain, crack plane elastic strain, crack plane inelastic (or fracture) strain vectors (3 × 1)
$\tilde{\boldsymbol{\varepsilon}}_c$	$\omega\tilde{\mathbf{e}}_c$
ζ	effective damage strain parameter
ζ_f	effective crack opening strain parameter
ζ_t	f_{t0}/E_m (tensile strain measure)
Z_0	initial value of fraction hardening parameter (Appendix B)
$Z(\kappa)$	friction hardening/softening function (Appendix B)
η, η_ε, σ_0	constant in the crack-bridging stress function
η_c	normalised plastic work hardening function
η_a, η_v	normalised fibre aspect ratio and fibre volume fraction
θ	lode angle (Appendix B)