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Probability-interval hybrid reliability analysis for cracked structures existing epistemic uncertainty

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ABSTRACT

The uncertainty modeling and reliability analysis for cracked structures in which there are many inherently uncertain parameters is very important in engineering. This paper presents a probability-interval hybrid uncertainty model and a corresponding efficient reliability analysis method for the structural cracking problem. Through introducing interval uncertainty, the method can effectively address the difficulties in the epistemic uncertainty modeling due to the lack of experimental samples, which expand greatly the applicability of reliability analysis technology in cracked structure research. The parameters are classified, and subsequently the probability and interval methods are separately applied to address the parameters with sufficient and insufficient experimental samples. A probability-interval hybrid reliability analysis model for the cracked structure based on the traditional first-order reliability method is developed. The scaled boundary finite element method is adopted to calculate the stress intensity factors from which the performance function can be obtained. Based on these calculations, an efficient iterative algorithm using the response surface is developed to solve the hybrid reliability model and calculate the interval of the failure probability of the cracked structure. Four numerical examples are presented for verification of the validity of the proposed method.

1. Introduction

Structural cracking is one of the primary causes of equipment damage, including such well known examples as the cracking of airframes, crack propagation in pressured pipelines and the fatigue fracture of locomotive rails and vehicle axles. In the analysis of cracked structures, there are many inherently uncertain parameters, such as the load, the material properties and the crack geometric characteristics. Therefore, the precise uncertainty modeling and reliability analysis and design to improve the security level of the cracked structure are very important in engineering. In recent years, a series of research in the field of the reliability analysis of cracked structures has been published. Rahman [1] developed a multivariate cracked structure approximate model based on engineering experience and the first-order reliability method (FORM) for the reliability of cracked tubular structures. Grigoriu et al. [2] applied first and second order reliability methods (FORM/SORM) to predict the confidence intervals in the initial direction of crack extension and the probability of fracture initiation. Rahman and Kim [3] developed a probabilistic methodology for fracture-mechanics analysis of nonlinear cracked structures. Peter [4] proposed a program calculating the failure probability and partial safety factors of cracked structures with uncertainty in

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Nomenclature

a	crack length
c_i	integration constant
$\mathbf{D}$	elastic matrix
F_{X_i}	cumulative distribution function of the random variable X_i
FORM	first-order reliability method
g	performance function
HL–RF	Hasofer–Lind and Rackwitz–Fiessler method
MPP	most probable point
MCS	Monte Carlo simulation
Pr	probability
P_f	failure probability
P_f^L, P_f^R	lower and upper bounds of the failure probability
K_I	mode-I stress intensity factor
K_{II}	mode-II stress intensity factor
K_{IC}	fracture toughness
K_{eff}	equivalent mode-I stress intensity factor
RS	response surface
r, θ	polar coordinates
SBFEM	scaled boundary finite element method
$\boldsymbol{\mu}_X$	mean vector of the random variables $\mathbf{X}$
$\mathbf{X}$	n-dimensional random vector
x, y	Cartesian coordinate
x_η, y_η	coordinates on boundary
$\mathbf{Y}$	m-dimensional interval vector
$\mathbf{Y}^c$	midpoint vector of the interval variables $\mathbf{Y}$
z	crack height
σ_{xx}, σ_{yy}, σ_{xy}	component stresses at any point of the crack tip region
σ^∞	far-field tensile stress
Φ	standard normal cumulative distribution function
$\boldsymbol{\Phi}_i$	deformation modes
α	crack angle
β^R, β^L	upper and lower bounds of the reliability index
θ_c	initial direction of crack propagation
ξ	radial coordinate
η	circumferential coordinate
κ_x, κ_y	sampling coefficients

the defect size. Rahman and Rao [5] used the virtual crack extension technique to calculate the sensitivities of the stress intensity factors and analyzed the reliability of elastoplastic cracked structures by Galerkin meshless methods. Reddy and Rao [6] presented a stochastic fracture mechanics analysis of elastoplastic cracked structures based on the continuum shape sensitivity analysis method in conjunction with the fractal finite element method (FFEM). Huang and Aliabadi [7] presented a probabilistic analysis of crack problems using a combination of the boundary element method (BEM) and FORM. Su and Zheng [8] adopted the spline fictitious boundary element method (SFBEM) and the response surface (RS) method to analyze the reliability of cracked structures. Chowdhury et al. [9] obtained a simulation solution of crack structural reliability by the Monte Carlo simulation (MCS).

In the above-mentioned works, the methods for the reliability analysis of cracked structures are usually based on probability theory and statistics, while all of the uncertain parameters, such as the load, material properties, fracture toughness and crack geometric characteristics, are treated as random variables with precise probability distributions. In the actual cracked structures, the precise probability distributions of some uncertainties are not difficult to determine under the conditions of convenient test methods and sufficient test samples, e.g., the applied loads, Young's Modulus and Poisson's ratio, etc. However, construction of the precise probability distributions for a portion of uncertain parameters can be difficult to achieve due to the epistemic uncertainty caused by the lacking of the text samples, and these parameters usually include the crack geometric characteristics, such as its size and angle. Epistemic uncertainty is subjective and reducible uncertainty that stems from lack of knowledge or data. Different theories based on non-probabilistic quantification have been developed to handle the epistemic uncertainty, which include evidence theory (or Dempster–Shafer theory) [10–13], probability-box [14–16], possibility theory [17,18], convex models [19,20], etc. When the traditional probabilistic methods are used to address such uncertain parameters, it has sometimes been necessary to assume their distribution functions. However, previous