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Finite block method for interface cracks

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ABSTRACT

The finite block method in both the Cartesian coordinate and polar coordinate systems is developed to evaluate the stress intensity factors and the T-stress for interface cracks in bi-materials. The first order partial differential matrices can be constructed straightaway based on the Lagrange series interpolation. The nodal values of displacement can be obtained from a set of linear algebraic equations in strong form from both the governing equation and the boundary conditions. In order to capture the stress intensity factors and the T-stress at the crack tip accurately, the asymptotic expansions of the stress and displacement around the crack tip are introduced with a singular core technique. For elastodynamic fracture problems, the Laplace transform method and the Durbin's inverse techniques are utilised. The accuracy and the convergence of the finite element method are demonstrated in three examples. Comparisons have been made with numerical solutions by using the boundary collocation method and the finite element method. Satisfactory numerical solutions are obtained with very few blocks in each example.

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1. Introduction

To determine the Stress Intensity Factors (SIFs) is a fundamental task for computational fracture mechanics in engineering. However, the second constant in the asymptotic expression of the crack-tip stress fields, named the T-stress, has to be considered in elastoplastic fracture mechanics. This is because the T-stress has certain effects on the crack growth direction, the shape and size of the plastic zone, the crack-tip constraint and the fracture toughness, etc (see [1–3]). Furthermore, in engineering constructions, it requires one material to be bonded to another such as adhesive joints, protective coatings, composite materials and thin films used in the manufacture of microelectronic circuits, etc. Dissimilar bi-materials or layered composites are often incorporated into a variety of components, such as smart structure sensors, actuators, and broadband magnetic probes. Having been recognised as one of the common failure modes of the general dissimilar bi-materials, the interface cracks could also be developed in the piezo-electro-magneto-elastic structures and thus affect the features of the electro-magneto-elastic apparatus. Unlike cracks in a body with homogeneous material, the stress intensity factors for the interface crack are coupled. Theoretical studies show that on the interface the stresses are of oscillatory behaviour of singularity and overlapping of crack surfaces at the crack tip.

In order to obtain the analytical solutions with partial differential equations in two-dimensions, the significant contributions towards understanding the physical and bi-material crack problem were made by Muskhelishvili [4], Williams [5], Sih and Rice [6], England [7], Erdogan [8], etc. As it is too difficult to obtain the analytical solution in closed form for engineering

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Nomenclature

Latin symbols
a crack length
$A^{(\alpha)}, B^{(\alpha)}, C^{(\alpha)}$ coefficients for different media
$\mathbf{D}_{,x}, \mathbf{D}_{,y}$ partial differential matrices
b_x and b_y body forces
c_0 characteristic speed
E, G Young's and shear modules
$F(\xi), G(\eta)$ shape functions in mapped domain
$H(t)$ Heaviside function
K number of sample in Laplace domain
$K_I + iK_{II}$ complex stress intensity factors
M, N seeds numbers
N_i shape function in physical domain
$n(n_x, n_y)$ normal to the boundary
(r, θ) polar coordinate system
r_0 size of singular core
R radius of disk
s_k Laplace parameter
t_0 characteristic time
T_0, σ free parameters of Laplace transformation
$T^{(\alpha)}$ T-stress
u_x^0, u_y^0, t_x^0 and t_y^0 boundary displacement and traction conditions
W width of plate
(x, y) coordinate in complex form
z complex

Greek symbols
$\varepsilon, \hat{\kappa}$ material parameters
Φ and Ψ stress functions of complex
Γ_u and Γ_σ displacement and traction boundaries
$\kappa^{(\alpha)}$ parameter of material
λ_n eigenvalues
$\mu^{(\alpha)}$ shear modulus
ρ mass density
$(\sigma_x, \sigma_y, \tau_{xy})$ stress tensor
(ξ, η) coordinate in mapping domain

problems, computational methods need to be developed. So far the Finite Element Method (FEM) is the most widely used computational method in engineering. Its applications in the interface crack problems can be found in references by Lin and Mar [9], Van et al. [10] and Hamoush and Ahmed [11] for two-dimensional problems. Since 1980s, the Boundary Element Method (BEM) has been an alternative method to obtain the solutions for the boundary value problems with some unique advantages, see Lee and Choi [12], Yuuki and Cho [13]. The M-integral technique with a properly selected auxiliary solution was proposed by Sladek et al. [14–19] to evaluate the T-stress for thermoelastic stress, elastodynamic stress and the interfaces using the BEM. It has been shown that the leading-order term dominance in the asymptotic expansion of stresses at the crack-tip vicinity is limited for elastoplastic behaving structures.

The Differential Quadrature Method (DQM) is a different kind of numerical technique for boundary value problems proposed by Bellman [20] in 1970. It was discovered that the DQM produced better convergent solutions than the FEM, when a similar number of discrete points/nodes are used. There has been comprehensive review of the DQM and its applications since the first paper was published by Bert and Malik [21]. To evaluate the stress intensity factors accurately for both static and dynamic cases, many new techniques have been developed recently. Extended finite element method was applied to dynamic cracks for piezoelectric solids by Bui et al. [22], Liu et al. [23], Bui [24] and Yu et al. [25]. The singular edge-based smoothed finite element method for stationary dynamic crack problems in 2D elastic solids was demonstrated by Liu et al. [26]. However, each numerical algorithm has its own advantages and disadvantages. In addition, the numerical modelling can be such complicated to be dealt with by using the finite element method, etc. For example, the fundamental

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