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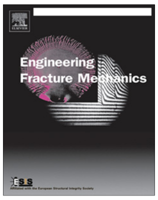

The beneficial effect of full or partial autofrettage on the combined 3-D stress intensity factor for an inner radial lunular or crescentic crack in a spherical pressure vessel

M. Perl *,[1,2], M. Steiner [1,3]

Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

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A B S T R A C T

The distributions of the combined 3-D Stress Intensity Factor (SIF), $K_{IN} = K_{IP} + K_{IA}$, due to both internal pressure and autofrettage along the front of an inner radial lunular or crescentic crack emanating from the bore of an overstrained spherical pressure vessel are evaluated. The 3-D analysis is performed using the finite element (FE) method employing singular elements along the crack front. A novel realistic autofrettage residual stress field incorporating the Bauschinger effect is applied to the vessel. The residual stress field is simulated using an equivalent temperature field in the FE analysis. SIFs for three vessel geometries ($R_0/R_i = 1.1$, 1.2, and 1.7), a wide range of crack depth to wall thickness ratios ($a/t = 0.01$–0.8), various ellipticities ($a/c = 0.2$–1.5), and three levels of autofrettage ($\varepsilon = 50\%$, 75%, and 100%) are evaluated. In total, about two hundred and seventy different crack configurations are analyzed. A detailed study of the influence of the above parameters on the prevailing SIF is conducted. The results clearly indicate the favorable effect of autofrettage in considerably reducing the prevailing effective stress intensity factor i.e., delaying crack initiation, slowing down crack growth rate, and thus substantially prolonging the total fatigue life of the vessel by up to twenty-fivefold. This favorable effect is found to be governed by σ_y/p – the ratio of the vessel's material initial yield stress to its internal pressure. The higher the ratio is, the more effective autofrettage becomes. Furthermore, the results emphasize the importance of properly evaluating the residual stress field due to autofrettage while at the same time accurately accounting for the Bauschinger effect, including re-yielding, as well as the significance of the three dimensional analysis herein performed.

1. Introduction

The process of autofrettage was suggested by Col. L. Jacob of the French Artillery [1] more than a hundred years ago for the purpose of increasing the allowable pressure in gun barrels, thus extending their firing range. Further on, it was found that the autofrettage process has an additional substantial benefit in decreasing the barrel's susceptibility to cracking, i.e., delaying crack initiation and slowing down crack growth rate, thus considerably increasing the total fatigue life of the barrel. This

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* Corresponding author. Tel./fax: +972 8 6477087.
E-mail address: merpr01@bgu.ac.il (M. Perl).

[1] Aaron Fish Professor and Dean Emeritus of Mechanical Engineering-Fracture Mechanics and Graduate student respectively.

[2] Fellow ASME.

[3] Presently, PhD student, Faculty of Aerospace Engineering, Technion Israel Institute of Technology, Haifa, Israel.

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Nomenclature

a	crack depth
A_p	Paris' constant
c	crack's half length
K_I	Mode I SIF
K_{IA}	Mode I SIF due to autofrettage
K_{IAmax}	maximum SIF due to autofrettage along crack front
K_{Ieff}	effective SIF
K_{Imax}	maximum SIF along crack front
K_{IN}	combined SIF
K_{INmax}	maximum combined SIF along crack front
K_{IP}	Mode I SIF due to internal pressure
K_{IPmax}	maximum SIF due to internal pressure along crack front
K_{IT}	Mode I SIF due to temperature field
$\tilde{K}_0$	normalizing SIF (Eq. (5))
n_p	Paris' constant
N	number of fatigue cycles
Q	shape factor for lunular or crescentic crack (Eq. (6))
P_y	the pressure at yielding onset of a non-autofrettaged spherical vessel (Eq. (4))
p	internal pressure
R_i	inner radius of the spherical vessel
R_0	outer radius of the spherical vessel
r, θ, φ	spherical coordinates
t	spherical vessel's wall thickness

Greek symbols

ε	level of autofrettage
η	percentile reduction in the effective SIF (Eq. (8))
ν	Poisson's ratio
σ_{rr}	radial stress component
$\sigma_{\theta\theta}$	hoop stress component
$\sigma_{\varphi\varphi}$	meridional stress component
σ_y	initial yield stress
ψ	parametric angle for lunular and crescentic cracks (1)
ψ_0	value of ψ at the cusp – the intersection of the crack front and the inner surface of the vessel

Acronyms

DOF	Degrees of Freedom
FEM	Finite Element Method
SIF	Stress Intensity Factor
SMP	Safe Maximum Pressure

Subscripts

Auto.	autofrettaged
non-Auto.	non autofrettaged

process has been further developed and has been widely used for cylindrical pressure vessels in a variety of industries for more than a century.

Spherical pressure vessels, though less common than cylindrical ones, are widely used in industry mainly due to their optimal specific strength (strength/weight) and their ease of packing. Some of these spherical pressure vessels [2] are manufactured from a series of double curved petals welded along their meridional lines. These vessels are susceptible to cracking along the welds due to various factors such as: cyclic pressurization–depressurization, the existence of a heat-effected zone near the welds, tensile residual stresses within this region, and the presence of corrosive agents. As a result a radial crack may develop from the inner surface of the vessel on the welding plane (Fig. 1a).

To date, autofrettage is rarely applied to spherical pressure vessels and its beneficial effect on such vessels has never been investigated. The fracture endurance, fatigue crack growth rate, and the total fatigue life of an autofrettaged spherical pressure vessel are all controlled by the largest prevailing effective SIF – K_{Ieff}, which in general may consist of three components: K_{IP} – the SIF due to internal pressure; K_{IA} – the SIF due to autofrettage residual stress field, and K_{IT} – the SIF resulting from temperature gradients. Presently, no values for K_{IT} in a spherical pressure vessel are available. However, since in the case of a typical modern gun barrel which is subjected to very high temperature transients, the three dimensional K_{IT} values are generally