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A three-node triangular element fitted to numerical manifold method with continuous nodal stress for crack analysis

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ABSTRACT

A three-node triangular element fitted to the numerical manifold method with continuous nodal stress called Trig3-CNS (NMM) element for accurately modeling two-dimensional linear elastic fracture problems is presented. By adopting two cover systems, namely, the mathematical cover and physical cover, the numerical manifold method (NMM) could easily solve continuous and discontinuous problems in a unified way. In contrast to the three-node triangular element (Trig3), the Trig3-CNS element has higher order of approximations, much better accuracy and continuous nodal stress. Moreover, it is free from the "linear dependence" which otherwise cripples many of the partition of unity based methods with high order approximations. The purpose of the present work is to synergize the advantages of both the recently developed Trig3-CNS element and the NMM to precisely model two-dimensional linear elastic fracture problems. A number of numerical examples indicate the accuracy and robustness of the present Trig3-CNS (NMM) element.

1. Introduction

The fracture behavior of cracked brittle structures, such as hard rocks, depends greatly upon the stress and strain in the vicinity of the crack tip. Accurate prediction of the singular stress fields near the crack tip using analytical, semi-analytical and experimental methods for complex engineering problems is very difficult. Therefore, many effective numerical methods such as the finite element method (FEM) [1], the smoothed finite element method (S-FEM) [2], the boundary element method (BEM) [3], the meshfree method [4], the extended finite element method (XFEM) [5] and the generalized finite element method [6] have been proposed to fulfill this task.

FEM have been used to simulate crack problems for several decades [7,8]. Application of FEM to such class of problems faces several difficulties relating to the need to construct a mesh which should conform to the crack faces, and to design substantially more refined mesh around the crack tip than in the remainder of the problem domain in order to obtain sufficiently accurate solution [5]. When considering the growth of cracks, the difficulties are further amplified, because then remeshing of the vicinity of the crack tips is inevitable. Additionally, it is well known that accuracy of some classic iso-parametric elements is highly sensitive to the quality of meshes [9]. However, it is difficult to avoid distorted elements in the simulation of fracture propagation which requires numerous remeshing tasks. In order to avoid tip-remeshing, the algorithm of edge

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Nomenclature

Ω_i^m	mathematical patch (MP)
n^m	number of mathematical patches
Ω	problem domain
Ω_{j-i}^p	j-th physical patch generated from the i-th mathematical patch Ω_i^m
n_i^p	number of physical patches that are generated from Ω_i^m
E_i	manifold element
$w_i(\boldsymbol{x})$	weight function
GN_{j-i}^p	generalized node of physical patch Ω_{j-i}^p
n^p	number of all the physical patches
$u_k(\boldsymbol{x})$	local approximation function
$\boldsymbol{d}_k$	array of unknown coefficients
$\boldsymbol{p}(\boldsymbol{x})$	matrix of polynomials bases
$\boldsymbol{F}^e(\boldsymbol{x})$	first items of Williams' displacement series
(r, θ)	polar coordinates with regard to the polar system defined at the crack tip
$u^h(\boldsymbol{x})$	global approximation
n^{pj}	number of physical patches for a manifold element E_j
L_i	area coordinate
$\boldsymbol{A}$	moment matrix
$\boldsymbol{B}$	basis matrix
$\boldsymbol{a}$	vector of nodal displacements
$\phi_k(\boldsymbol{x})$	shape function corresponding to physical patch Ω_k^p or node k
$n^{[i]}$	number of supporting nodes for physical patch Ω_i^p
$\sigma_{ij}^{\text{real}}$	stress tensor corresponds to the actual state
$\varepsilon_{ij}^{\text{real}}$	strain tensor corresponds to the actual state
u_i^{real}	displacement vector corresponds to the actual state
σ_{ij}^{aux}	stress tensor corresponds to the auxiliary state
$\varepsilon_{ij}^{\text{aux}}$	strain tensor corresponds to the auxiliary state
u_i^{aux}	displacement vector corresponds to the auxiliary state
$I^{(\text{real, aux})}$	interaction integral
$W^{(\text{real, aux})}$	interaction strain energy
R	domain radius
h	size of mathematical patch
R_d	factor which can determine the size of domain radius R
$q(\boldsymbol{x})$	bounded weighting function
n	total number of the nodes in the computational model
e_{d}	displacement norm
e_{e}	energy norm
$\boldsymbol{u}^{\text{ex}}$	exact or analytical displacement vector solution
$\boldsymbol{u}^{\text{num}}$	numerical displacement vector solution
ε^{ex}	exact or analytical strain vector solution
ε^{num}	numerical strain vector solution
$\boldsymbol{D}$	elastic matrix
E	Young's modulus
v	Poisson's ratio
M	bending moment
I	moment of inertia
P	traction
a	length of crack
K_{I}	stress intensity factor corresponding to mode I
K_{II}	stress intensity factor corresponding to mode II
M_{I}	normalized stress intensity factor corresponding to mode I
M_{II}	normalized stress intensity factor corresponding to mode II

rotation for computational fracture has been proposed in the framework of FEM [10–12]. S-FEMs [2] have been developed recently by Liu and his co-workers to improve accuracy of FEM. In the simulation of crack problems, S-FEMs also need to construct a mesh conforming to the crack faces which will hinder the application of S-FEMs to complex crack propagation