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Short Communication

Instability of interfaces of gas bubbles in liquids under acoustic excitation with dual frequency

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ABSTRACT

Instability of interfaces of gas bubbles in liquids under acoustic excitation with dual frequency is theoretically investigated. The critical bubble radii dividing stable and unstable regions of bubbles under dual-frequency acoustic excitation are strongly affected by the amplitudes of dual-frequency acoustic excitation rather than the frequencies of dual-frequency excitation. The limitation of the proposed model is also discussed with demonstrating examples.

1. Introduction

Instability of bubble interfaces in liquids is of great importance to both the fundamental and applied physics of cavitation and bubble dynamics [1–19]. A thorough understanding of this topic is essential for many paramount problems with practical applications, e.g., predictions of bubble sonoluminescence [7,8], bubble growth through mass transfer [4,13,20–23], under water explosions [24], rise velocity of bubbles [14,15], cavitation associated damage to propeller [25,26] and medical diagnostic ultrasound imaging [16]. In reality, asymmetries of the bubble shape can be induced due to various reasons e.g., pressure gradient across bubbles, presence of other bubbles or interfaces and gravity. For example, shape oscillations of bubbles can be parametrically excited by the bubble pulsation under a sound field. The study of surface instability provides the validity of the generally adopted assumption of spherical bubbles [4,21–23,27–29] against paramount parameters (e.g., bubble radius, frequency and amplitude of acoustic excitation). Specifically, for chemical engineering, the study of shape instability can help facilitate the sonochemistry process, design effective sonochemical reactor and improve the efficiency of the whole system.

In present paper, a brief summary of the previous works on the instability of bubble surfaces in the literature is given. For classical review papers on this topic, readers are referred to Plesset and Prosperetti [1] and Feng and Leal [2]. Instability of bubble interfaces has been theoretically studied by many researchers e.g., Plesset [3], Eller and Crum [6], Prosperetii [5], Brenner et al. [7], Shaw [9,10] and Harkin et al. [19]. Plesset [3] established a well-known framework to study the fluid flows with spherical symmetry (e.g., bubbles). In Plesset [3], the small departure from spherical form is expressed in terms of an infinite sum of spherical harmonics. Hsieh and Plesset [4] followed Plesset's analysis to predict the threshold of shape instability of a growing bubble. The inclusion of viscous effects was done by Proseperetti [5] and experimental measurement of threshold of shape instability was provided by Eller and Crum [6]. Brenner et al. [7] distinguished two different mechanisms (Rayleigh–Taylor instability and parametric instability respectively) leading to deviation of bubble interfaces from spherical shapes. The existence of several striking phenomena (e.g., bubble sound emission and erratic bubble dancing problem) provided a strong impetus on the study of coupling between different shape modes, interactions between shape distortions and translational motions, and energy transfer between shape and volume modes. Shaw [9,10] studied the nonlinear mutual interactions

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between the axisymmetric shape oscillations, the axial translational motion, and the volume oscillations of the gas bubble in the liquid. Harkin et al. [19] studied the bidirectional transfer of oscillation energy between shape and volume modes.

Recently, bubble dynamics under acoustic excitation with multiple frequencies (e.g., dual and triple frequency) has been intensively investigated by researchers [23,30–39] with the background of sonochemistry and bubble sonoluminescence. Comparing with bubbles excited by single-frequency acoustic waves, cavitation effects are much stronger when bubbles are induced by multi-frequency acoustic waves. For example, it was found that the bubble sonoluminescence under dual-frequency excitation can be boosted up to 3 times of those under single-frequency excitation [33]. Pandit and collaborators conducted a series of work both theoretically and experimentally investigating cavitation and bubble dynamics within multi-frequency sonochemical reactor [32,33]. Feng et al. [30] found that cavitation yield can be greatly enhanced by multi-frequency ultrasonic irradiation. Moholkar et al. [40] theoretically studied the influences of several paramount parameters (e.g., frequency and amplitude ratio, phase difference between waves) on the bubble dynamics and spatial distribution of cavitation events in a dual-frequency ultrasonic processor. A more advanced model was further developed by Moholkar et al. [41] to understand energy transformation chain in ultrasonic processor. Moholkar [42] optimized the dual-frequency ultrasonic processor numerically based on spatially averaged energy transmitted and spatial uniformity of the acoustic pressure amplitude. Recently, Zhang [23] studied the effects of mass transfer on the oscillations of bubbles induced by dual-frequency excitation and found that there exists a threshold of acoustic pressure amplitude for enhancing cavitation effects when using dual-frequency acoustic excitation.

From literature review, it was found that the assumption of spherical bubbles is widely adopted when studying bubble dynamics under dual-frequency excitation while the valid regions of this assumption have not been shown yet. In present paper, instability of gas bubble interfaces oscillating in liquids under dual-frequency acoustic excitation is analytically studied following a well-known Plesset's framework using spherical harmonics to represent shape distortions. An expression of critical bubble radius dividing stable and unstable oscillating regions is derived and influences of paramount parameters (e.g., acoustic pressure amplitude) are shown with several demonstrating examples together with valid regions of present work.

2. Theoretical analysis

Here, we assumed that the bubble interface has a distortion from the spherical shape with bubble radius R as follows,

$$r_s = R + \sum_{n=1}^{\infty} a_n Y_n. \tag{1}$$

Here r_s is the vector of bubble interface; R is the radius of aforementioned spherical bubble interface; Y_n is nth order spherical harmonics; a_n is one of a series of unknown coefficients and is assumed to be independent with each other. For convenience, we assumed that the two fluids inside and outside bubbles are immiscible, nonviscous and incompressible and also assumed that $|a_n| \ll R$.

Based on Plesset's analysis [3], one can obtain the differential equation of a_n:

$$\frac{d^2 a_n}{dt^2} + \frac{3}{R}\frac{dR}{dt}\frac{da_n}{dt} - Aa_n = 0, \tag{2}$$

with

$$A = \frac{[n(n-1)\rho_l - (n+1)(n+2)\rho_g]d^2R/dt^2 - (n-1)n(n+1)(n+2)\sigma/R^2}{[n\rho_l + (n+1)\rho_g]R}. \tag{3}$$

Here t is the time; n is the order of shape oscillations of bubbles; σ is the surface tension coefficient; ρ_l is the density of the liquid outside the bubbles; ρ_g is the density of the gas inside the bubbles.

For the problems of gas bubbles oscillating in liquids to be discussed in present paper, it is safe to assume that $\rho_g \ll \rho_l$. Then Eq. (3) reduces to [3]:

$$A = \frac{(n-1)}{R}\frac{d^2R}{dt^2} - (n-1)(n+1)(n+2)\frac{\sigma}{\rho_l R^3}. \tag{4}$$

For convenience, Eqs. (2) and (4) are transformed into [4]:

$$\frac{d^2 b_n}{dt^2} + Gb_n = 0, \tag{5}$$

with

$$b_n = R^{3/2} a_n,$$

$$G = (n-1)(n+1)(n+2)\frac{\sigma}{\rho_l R^3} - \frac{3}{4R^2}\left(\frac{dR}{dt}\right)^2 - \frac{(n+\frac{1}{2})}{R}\frac{d^2R}{dt^2}. \tag{6}$$

With viscosity and liquid compressibility ignored, the equation of bubble wall can be described by [1]:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_{ext}(R,t) - p_s(t)}{\rho_l}, \tag{7}$$

where

$$p_{ext}(R,t) = \left(P_0 + \frac{2\sigma}{R_0}\right)(R_0/R)^{3\kappa} - \frac{2\sigma}{R}, \tag{8}$$

$$p_s(t) = P_0 + P_{A_1}\cos(\omega_1 t) + P_{A_2}\cos(\omega_2 t). \tag{9}$$

Here κ is the polytropic exponent; P_0 is the ambient pressure; R_0 is the equilibrium bubble radius; P_{A_1} and P_{A_2} are the amplitudes of acoustic excitation with angular frequencies ω_1 and ω_2, respectively. In present paper, for convenience, we assumed that P_{A_1} and P_{A_2} are of the same order and $\omega_1 \geqslant \omega_2$.

Ignoring the high order terms (e.g., harmonics, excitation with sum and difference of two frequencies), the solution of Eqs. ((7)–(9)) is [23]:

$$R = R_0[1 + \delta_1\cos(\omega_1 t + \phi_1) + \delta_2\cos(\omega_2 t + \phi_2)], \tag{10}$$

with

$$\delta_1 = -\frac{P_{A_1}}{\rho_l R_0^2|\omega_0^2 - \omega_1^2|}, \tag{11}$$

$$\delta_2 = -\frac{P_{A_2}}{\rho_l R_0^2|\omega_0^2 - \omega_2^2|}, \tag{12}$$

$$\omega_0^2 = \frac{1}{\rho_l R_0^2}\left[3\kappa\left(P_0 + \frac{2\sigma}{R_0}\right) - \frac{2\sigma}{R_0}\right]. \tag{13}$$

Here ω_0 is the natural frequency of gas bubble oscillations in liquids. The expressions of the phases ϕ_1 and ϕ_2 are not used in present paper hence their expressions are not given here. For details, readers are referred to Zhang [23].

Substituting Eq. (10) into Eq. (6) and omitting terms with order higher than the first order, one can obtain