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A unified finite element method for the simulation of hydraulic fracturing with and without fluid lag

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ABSTRACT

Hydraulic fracturing with and without fluid lag has different flow boundary conditions at the fluid front, which always results in different simulation methods. In this paper, we extend a finite element method (Bao et al., 2015) to simulate hydraulic fracturing with and without fluid lag in a unified manner. A unified numerical boundary condition is imposed on the fluid front independent of fluid lag situations. No effort is needed to track the fluid front explicitly, and the burden of model re-meshing induced by fluid front advancement is avoided. The method is verified by comparing numerical simulations with some analytical solutions. The simulations cover hydraulic fracturing with constant fluid lag fraction, without fluid lag, and with vanishing fluid lag. Some factors governing the simulations are discussed.

1. Introduction

Hydraulic fracturing is defined as the process where the propagation of a fracture is driven by the injection of pressurized fluid into the host solid medium [2]. Hydraulic fracturing can be found in natural occurrences such as the formation of the magma-driven dike [3,4], and the growth of fracture along glacier beds driven by water [5]. Actually, hydraulic fracturing has been accepted as an important technique to improve the recovery of conventional and unconventional petroleum and natural gas resources in deep strata, and to remediate waste [6] and induce cave in mining near a free surface [7].

A gap zone between the fluid front and the fracture tip may exist in hydraulic fracturing. This gap zone is referred to as fluid lag. The existence of a fluid lag near the fracture tip has been realized since 1950s [8–10]. In some cases the fluid lag zone is a clear one [11], while in some other cases fluid flow from the solid medium into the fracture is inferred from experiments when the solid medium has high permeability [12]. Fluid lags have huge impact on hydraulic fracturing [13,14]. A couple of investigators discussed the factors that govern fluid lags. Garagash [15] discussed the effect of energy dissipation regimes on fluid lags when there is no confining stress. It is found in his discussion that a fluid lag exists and plays an important role when the fracture propagation regime [16] is viscosity-dominated, and it does not exist when the fracture propagation regime is toughness-dominated. Lecampion and Detournay [11] discussed the effect of confining stress on fluid lag evolution, and discovered that a fluid lag tends to vanish when the medium is applied with non-zero confining stress. These investigations imply that in deep strata a fluid lag may gradually vanish following its appearance and extension, and in shallow subsurface a fluid lag may appear and extend even when the fracture is completely filled by the fluid on

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Nomenclature

$\boldsymbol{B}$	to transfer pressure into equivalent node force
$\boldsymbol{C}$	equivalent node force of confining stress
$\boldsymbol{D}$	elastic stiffness tensor
e_l	relative error of half fracture length
$e_{p(0)}$	relative error of fluid pressure at the injection point
$e_{w(0)}$	relative error of fracture width at the injection point
e_{ξ_f}	relative error of ξ_f
E	elastic modulus
E'	modified elastic modulus
$\boldsymbol{F}$	equivalent global nodal force of confining stress
$\boldsymbol{H}$	to account for the contribution of fluid boundary conditions
K_I	stress intensity factor
K_I^u	upper limit of K_I
K_{IC}	fracture toughness
K_m	dimensionless fracture toughness
K'	modified fracture toughness
$\boldsymbol{K}$	to relate fracture with to fluid pressure
$\boldsymbol{K}_u$	global stiffness of the solid elements
$\boldsymbol{K}_w$	the assembly of the flux stiffness of the fluid elements
l	half fracture length
l_e	expected half fracture length
l_N	half fracture length obtained by the numerical method
l_S	half fracture length in the self-similar solutions
$\boldsymbol{L}$	the assembly of the length stiffness of the fluid elements
$\boldsymbol{n}$	outward unit normal of fracture
p	net pressure
p_c	characteristic fluid pressure
p_f	fluid pressure
p^l	lower limit of p_s
$p_{\max}$	the maximum pressure in the fracture
p_s	fluid pressure at fluid front
p^u	upper limit of p_s
p_{s_n}	fluid pressure at fluid front at the end of the nth step
$\boldsymbol{P}_f$	a vector formed by the fluid pressure on fracture surface
$\boldsymbol{P}_{n+1}$	the fluid pressure vector at the $(n+1)th$ step
$\boldsymbol{q}$	fluid flux
Q_0	injection rate
s	half fluid length
Δs	fracture segment length ahead of fluid front
t_c	characteristic time
Δt	time step
Δt^i	initial guess of time step
w	fracture width
w_c	characteristic fracture width
$\boldsymbol{W}$	a vector formed by node width on the fracture surface
$\boldsymbol{W}_{n+1}$	the fracture width vector at the $(n+1)th$ step
$\boldsymbol{u}$	displacement
$\boldsymbol{U}$	global nodal displacement
$\bar{V}$	mean fluid velocity at fluid front
u_x, u_y	displacement in x, y direction, respectively

Greek letters

δp	allowable testing function
∂S	collection of boundary conditions
σ_0	confining stress
$\boldsymbol{\sigma}$	stress
$\boldsymbol{\varepsilon}$	strain
ε_p^l	lower limit coefficient of p_s
ε_p^u	upper limit coefficient of p_s