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Transient problem for semi-infinite crack in periodically voided materials

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ABSTRACT

A method, which has been recently established for the modeling of long (semi-infinite) cracks embedded in materials with periodic microstructure, is implemented to determine the initial static and dynamic field in materials in which square voids are distributed in a doubly periodic manner. The method is based on the utilization of the K-field of the homogeneous orthotropic material that effectively represents the voided material far away from the crack. The Mode I field distribution is determined by solving the governing equations and boundary conditions, in conjunction with the discrete Fourier transform. The occurrence of the overshooting phenomenon is investigated for various values of the parent solid material volume fraction and the possibility of a stable crack propagation is determined.

1. Introduction

Recent advances in manufacturing of voided materials with the desired periodic microstructure stimulated studies of their fracture behavior. A number of works have been dedicated to the prediction of fracture toughness of cellular materials (e.g., [8]), as well as materials of low and medium porosity, Lipperman et al. [12], Ryvkin and Aboudi [14].

The dynamic fracture phenomena in periodic materials received significantly less attention. A comprehensive discussion of the results obtained in the framework of lattice models can be found in Slepyan [19–21]. In particular, Slepyan [19] has examined the dynamic overshoot phenomenon in elastic and viscoelastic mass-spring systems. Note, that for sudden (Heaviside-type) excitation and for an elastic bonds the overshooting in this system always takes place.

The dynamic response of two-dimensional porous elastic materials in which square voids are distributed in a doubly periodic manner has been recently investigated by Aboudi and Ryvkin [3]. In particular, the overshooting was examined by the analysis of a sudden break of an intervoid ligament in an initially undamaged material subjected to a remote tensile loading, and it was found that this phenomenon manifested itself only in the case of sufficiently high material relative density. An essential next step in this topic is the investigation of overshooting for the case of a long (semi-infinite) crack advance in periodically voided material which is the subject of the present paper.

In order to investigate the transient response in the case of a long crack, the initial elastostatic field in the cracked material must be established. The initial condition for the considered transient problem is a self-similar static stress K-field of a semi-infinite crack. A conventional approach to derivation of this field is by a direct modeling of a sufficiently large periodic domain enclosing the crack tip with boundary conditions at the remote boundaries which corresponds to

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Nomenclature

a	half of the intervoid ligament thickness
$\boldsymbol{C}(C_{jkpq})$	stiffness tensor for solid(parent) material
C_p, C_t	longitudinal and shear wave speeds, respectively (m/s)
D_{jkpq}	4-th order damage tensor (1)
D_j	damage tensor components for isotropic material (1)
E	Young modulus of solid material (MPa)
H, L	half of the height and half of the width of the rectangular analysis domain (m)
h, l	half of the height and half of the width of the representative cell, respectively (m)
I_{jkpq}	4-th order unit tensor (1)
$\bar{J}^{\sigma}_{ij}, \bar{J}^{\sigma(K_q)}_{ij}$	stress jumps between the opposite boundaries of rectangular domain for K-field in global and local systems of coordinates, respectively (MPa)
$\bar{J}^{u}_{ij}, \bar{J}^{u(K_q)}_{ij}$	displacement jumps between the opposite rectangular domain boundaries for K-field in global and local systems of coordinates, respectively (m)
$J^{\sigma(K_q)}_{ij}$	stress jumps between the opposite boundaries of rectangular voided domain (MPa)
$J^{u(K_q)}_{ij}$	displacement jumps between the opposite boundaries of rectangular voided domain (m)
K_I	Mode I stress intensity factor ($N/m^{3/2}$)
$K_{IC}, \bar{K}_{IC}$	Mode I fracture toughness of voided material ($N/m^{3/2}$)
$\bar{K}_{IC}$	normalized Mode I fracture toughness of voided material (1)
K_2, K_3	integer coordinates of cell location (1)
M_2, M_3	integer numbers defining the number of cells in rectangular domain (1)
r	polar coordinate measured from the crack tip (m)
$\boldsymbol{S}^*(S_{ij})$	effective compliance tensor (1/MPa)
s_j, β_j	imaginary parts of the roots and their absolute values respectively (1)
t	time (s)
$T^{(K_2,K_3)}_{ij}$	tractions at cell boundaries (MPa)
$u_j, u^{(K_2,K_3)}_j$	displacements in elastodynamic field in global and local coordinate systems (m)
$\bar{u}_j$	displacements in K-field solution for anisotropic homogeneous plane (m)
p_j, q_j	service variables defining the displacements field (1/MPa)
v_s	volume fraction of the solid phase in the voided material
X'_2, X'_3	local coordinate system in repetitive cells (m)
X_2, X_3	global coordinate system (m)
μ_j	roots of characteristic Eq. (1)
α_j	real parts of the roots of characteristic Eq. (1)
β, γ	subcell labels
η_j	weight coefficients in stress jump boundary condition (1)
θ	polar angle for the crack tip coordinate system
$\bar{\sigma}_{ij}$	stresses in K-field solution for anisotropic homogeneous plane (MPa)
σ^0_{ij}	initial elastostatic in voided material (MPa)
$\boldsymbol{\sigma}(\sigma_{ij})$	dynamic stress field tensor (MPa)
σ^{0e}_{ij}	eigenstress tensor in voided material (MPa)
σ_{eq}	equivalent stress (MPa)
ϵ^0_{pq}	elastostatic strain tensor (1)
σ_f	tensile strength of parent solid material (MPa)
$\sigma^S_{22}, \sigma^D_{22}$	elastostatic and elastodynamic stress field components, respectively
ϕ_p, ϕ_q	Fourier transform parameters (1)
ρ	mass density of solid material

the K-field for a crack in homogeneous material possessing effective elastic properties. This approach is widely used in the study of cellular materials (e.g., [6,7,22]) which can be modeled as beam lattices. A number of studies of cracks in periodically layered materials are also based on this method, e.g., Jha and Charalambides [9] and Ballarini et al. [5]. However, in the present case of 2D continuum with square voids of arbitrary size it becomes impractical due to the large number of degrees of freedom that must be employed. Therefore, a novel method for derivation of the near-tip field suggested recently by Ryvkin and Hadar [15] is utilized. At the first step, as in the conventional approach, the K-field for the homogeneous orthotropic material with effective elastic properties must be determined. These properties are presently derived by the high-fidelity generalized method of cells (HFGMC) micromechanical method, see Chapter 6 of Aboudi et al. [4]. The obtained K-field is subsequently employed for the formulation of the conditions at the boundaries of rectangular domain around the crack tip in terms of jumps of the displacement and traction components. This allowed the application of the representative cell