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Analytical fractal model for rugged fracture surface of brittle materials

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ABSTRACT

The fractal modeling of a rugged fracture surface has received different purposes. However none definitive model for the most of materials has been reached. Therefore, a general self-affine fractal model is proposed for fracture surfaces and applied to heavy clay and mortar. An analytical expression for the rugged crack length is obtained for application on fractal fracture mechanics. Stereoscopic images are obtained for each tested specimens. Image processing filters are used to extract the rugged profile of the cracks. The box-counting and sand-box methods are used on the crack profile to obtain the local and the global roughness exponents. Specimens prepared under different conditions validated the model. Mortars and heavy clay specimens were characterized by measuring their modulus of rupture and the rugged crack profile under 3-point bending tests. A good agreement between the model and the experimental results was observed. A strong correlation between the fractal dimension and the sintering temperature for heavy clay specimens was verified. The results also showed that the increasing rugged crack length of the profile of the fractured mortar specimens is well correlated with the increase in water/cement ratio. These results validate the application of the proposed model for estimating the fracture strength of brittle materials.

1. Introduction

The fracture surface is a record of the information left by the fracture process. Generally, the rugged fracture surface profile has fractal geometry, so it is possible to establish a relationship between its topology and physical quantities of the fracture mechanics using fractal characterization techniques. However, the classical fracture mechanics (CFM) was developed idealizing a flat, smooth, and regular fracture surface, as the geometry of crack surfaces is usually rugged and cannot be easily described by the Euclidean geometry [1]. In this sense, the mathematical basis of CFM considers an energetic equivalent between the rugged and the projected fracture surfaces [2]. Besides the mathematical complexity, part of this

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Nomenclature

Latin characters

0 index to denote measurements taken on the projected plane
a minimal area of fracture surface
ΔA area of fracture surface
$A1, A2, A3; Ar1, \ldots, Ar30$ mortar and ceramic samples
$B1, B2, B3; Br1, \ldots, B30$ mortar and ceramic samples
$C1, C2, C3; Cr1, \ldots, Cr30$ mortar and ceramic samples
d infinitesimal increment
$D1, D2, D3; Dr1, \ldots, Dr30$ mortar and ceramic samples
D_B fractal box dimension
D_f fractal dimension
D_D divider dimension
D_{BC} fractal dimension measured by box-counting method
e_0 fractal cell size or minimal crack length in transversal direction
E elastic or Young' s modulus
E_0 crack length measured in transversal direction
ΔE_0 variation of crack length measured in transversal direction
f general functions
g general functions or subscript index for global quantity
G elastic energy release rate
h_0 fractal cell size or minimal crack length in vertical direction
H Hurst's exponent
H_0 plane projected crack height
ΔH_0 variation in the plane projected crack height
K multiplicative factor
K_x multiplicative factor in x-direction
K_y multiplicative factor in y-direction
l subscript index for local quantity
L rugged crack length
L_{BC} rugged crack length measured by box-counting method
l_x, l_y, l_z rulers length or scale in the x-direction, y-direction, z-direction, respectively
l_0 fractal cell size or minimal crack length in propagation direction
L_0 plane projected crack length measured in propagation direction
L_{0S} saturation plane projected crack length measured in propagation direction
L_{0C} Griffith's critical crack length
ΔL rugged crack length
ΔL_0 distance between two points of the crack (the projected length of the crack)
ΔL_{0C} critical crack length
N_L number of units of the crack length in longitudinal or propagation direction
N_T number of units of the crack length in transversal direction
N_V number of units of the crack length in vertical or height direction
N_x number of units of the crack length in the growth direction
N_y number of units of the crack length in the perpendicular direction
sat subscript index indicate saturation
Y_0 shape function
x, y, z spatial coordinates
$\Delta x, \Delta y, \Delta z,$ incremental measure length in x-direction, y-direction, z-direction
$\Delta z_x, \Delta z_y$ fluctuation or roughness mean squares or incremental measure of crack height in x-direction, y-direction, respectively
w width of specimen test

Greek letters

β^* exponent
δ incremental measure
∂ partial derivative
$\varepsilon_L, \varepsilon_T$ horizontal and vertical scale of the fractal scaling
γ_e, γ_p elastic and plastic specific energy surface, respectively
λ_x amplification factor scale in x-direction