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The mode I crack–inclusion interaction in orthotropic medium

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ABSTRACT

The interaction between a mode I crack and a symmetrical shape inclusion in an orthotropic medium subjected to the remote stress is investigated. Based on the transformation toughening theory and the Eshelby inclusion method, a closed-form formula for the change of stress intensity factor near the crack tip induced by the inclusion has been proposed. Both the stiffness and the shape for the inclusion are studied by finite element analysis to validate the developed formula. The elastic properties along the orientation perpendicular to the crack line play more influence on the change of stress intensity factor than those in the other directions.

1. Introduction

Study on the crack–inclusion interaction plays an important role on the understanding of the mechanism of strength, fatigue, damage and fracture of materials. The mechanical properties of the medium, the shapes and the sizes of the inclusions have strongly affected on the interaction.

There have been numerous investigations on the interaction between inclusions and cracks in isotropic medium in literatures [1–15]. Based on the Muskhelishvili technique, Tamate [1] studied the variation of the stress intensity factor (SIF) for a circular inclusion ahead of a crack in an infinite plate. The inclusion geometries and the elastic properties for the plate were analyzed. Cheeseman and Santare [2] examined the interaction between a curved crack and a circular inclusion based on the Kolosove–Muskhelishvili stress potential technique. They indicated that the curvature for the crack as well as the inclusion stiffness played an important effect on the variation of SIF near the crack tip. In the literature [3], a crack was considered as distributed dislocations to investigate the elastic crack–inclusion interaction in isotropic materials. Yi et al. [4] applied the distributed dislocation technology on the interaction of a circular inclusion and a radial crack in an elastic–plastic matrix. They found that both the crack orientation and the gap between the crack and the inclusion influence on the SIF, the plastic zone size, and the crack tip opening displacement. Zhang et al. [5] studied the interaction of a crack and a circular inclusion in a finite plane by considering the inclusion as distributed dislocations. The Eshelby equivalent inclusion method [6] was used to obtain the equivalent transformation strain, and a closed-form formula for the difference of the SIF near a crack tip was developed by Li and his fellows [7–11]. Knight et al. [12] found that the Poisson ratio between an inclusion and medium played an important influence on the crack trajectory for an uncoated inclusion, and the crack-tip behaviors were changed distinctively for a coated inclusion. In the literatures [13,14], the complex variable method was adopted to solve the Eshelby's problem for an inclusion in an infinite matrix. By using the boundary element method, the effects on the dynamic SIF due to the inclusion position, material combinations and multiple micro-cracks, were discussed [15].

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Nomenclature

A	area for the inclusion
$\boldsymbol{a}_i$, p_i	eigenvectors and eigenvalues
a	crack length
b	width for the medium in finite element model
$\boldsymbol{C}_i$, $\boldsymbol{C}_m$	elastic tensors for the inclusion and the medium
c_{ijkl}	stiffness coefficients
c	height for the medium in finite element model
dA	infinitesimal area for the inclusion
E_i, E_m	Young's modulus for the inclusion and the medium
$\boldsymbol{e}^T$	equivalent transformation strains
$\boldsymbol{e}^A$	strain tensor near the crack tip
$f(z)$	arbitrary function of z
$\boldsymbol{I}$	unit tensor
J_{tip}	J-integral near the crack tip
K_{I}, K_{I}^{tip}	stress intensity factor and effective stress intensity factor
L, W	length and width for the rectangle inclusion
R	radius for the circular inclusion
$\boldsymbol{S}$	Eshelby tensor
s_{pqmn}	flexibility coefficients for material
u_k	displacement components
x'_1, x'_2	directions for the principal strains
z_i	location for the inclusion under global coordinate system
ΔK	The difference between the effective stress intensity factor and the stress intensity factor
ΔK_{I}, ΔK_{II}, ΔK_{III}	change for stress intensity factors for mode I, II, III crack
ε^T	trace for the transformation strains
$\varepsilon_{x'_1}$	principal strain respect to the global x_1 coordinate axis
ψ	angel of the axis for the principal strain $\varepsilon_{x'_1}$
λ	ratio for elastic properties between the inclusion and the medium
ν	Poisson ratio
σ_∞	remote stress
σ_{mn}, e_{pq}	stress and strain components

For the crack–inclusion interaction in anisotropic medium, the numerical method, such as the finite element method (FEM) and the boundary element method (BEM), is the regular and powerful technique [16]. In the literature [17], FEM was conducted to study the energy release rate of a crack propagating into a circular anisotropic inclusion, as well as the variations of the energy release rate due to the inclusion properties. By FEM, Gao et al. [18] investigated the effect on the SIF near a crack tip due to the properties of an inclusion in superconductor under electromagnetic load. Dong and Lee [19] studied the SIF near a crack tip in an anisotropic elastic medium with a circular inclusion by BEM. Theoretically, it is difficult to obtain a closed-form equation for the SIF variation near a crack tip if an inclusion is embedded in an anisotropic medium. The reason is that the solution for the sextic eigenvalue equation in the Stroh formula for the anisotropic material is hard to be achieved [20]. However, if a typical coordinate system was selected, the SIF variation near the crack tip can be obtained [21–22]. The coordinate system is that the symmetry axis was chosen as the direction of x_3 axis, and the plane based on the other x_1, x_2 directions was selected for the solution of sextic eigenvalue equation. On this condition, the material properties were identical along both directions in the plane, and were different along the other x_3 direction.

In present study, the interaction of a crack and an inclusion embedded in an infinite orthotropic medium is investigated. The coordinate system is that the symmetry axis is chosen in the x_1–x_2 plane, which is not the same as in literature. It is noticed that the mechanical properties in the plane are different. At first, the mismatched strains between the inclusion and the medium are considered as the transformation strains to develop a closed-form formula to predict the variation of the SIF near a crack tip. The Stroh theory and the Eshelby equivalent method are contributed in the derivation. Then, the approximate formula is validated by the finite element method. Finally, the influences on the SIF change due to the difference of the mechanical properties along x_1 and x_2 directions in the plane are discussed.

2. Problem definition

A semi-infinite crack is embedded in an infinite orthotropic elastic medium subjected a remote tensile stress (σ_∞) perpendicular to the crack plane. A symmetrical shape inclusion located near the crack as shown in Fig. 1. The plane strain condition is assumed in the analysis. The inclusion area is A, and dA is an infinitesimal area of the inclusion. The size of inclusion