Engineering Fracture Mechanics 136 (2015) 213–225

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

A constrained intrinsic cohesive finite element method with little stiffness reduction for fracture simulation

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ARTICLE INFO

Article history:
Received 18 June 2014
Received in revised form 24 December 2014
Accepted 29 January 2015
Available online 9 February 2015

Keywords:
Intrinsic cohesive finite element method
Constraint algorithm
Stiffness reduction
Dynamic fracture simulation
Extrinsic cohesive finite element method

ABSTRACT

To solve the stiffness reduction problem caused by cohesive surfaces in the intrinsic cohesive finite element model (CFEM), a series of constraint conditions are introduced into the algebraic equation system of the CFEM to constrain the separation of cohesive surfaces. To make the constrained cohesive surfaces active at the appropriate time, the constraint conditions will be removed in the high stress zone where fracture could initiate and propagate. The efficiency of the constrained CFEM, instead of being reduced by the constraint conditions, is significantly improved because the constrained cohesive surfaces are not counted in the numerical implementation.

1. Introduction

The cohesive finite element model (CFEM) pioneered by Needleman and coworkers [1–3] is an efficient approach to simulate fracture. In this method, a set of cohesive surfaces is embedded along all element boundaries. The bulk element is characterized by a volumetric constitutive relation, while the cohesive surface element is characterized by a cohesive law. The fracture is allowed to initiate and propagate only along the prescribed cohesive surfaces. The fracture process occurs along arbitrary paths, so any pattern can be explicitly simulated without a separate fracture criterion and mesh modification. Due to these advantages, the CFEM has been extensively applied to dynamic fracture simulation.

However, a CFEM with an initially elastic cohesive law, also called an *intrinsic* CFEM, has a critical drawback that is the material stiffness is significantly reduced as the density of the cohesive surfaces increases. This leads to a competing requirement on the element size. On one hand, to accurately simulate the fields in the crack tip vicinity, the element size must be small enough. On the other hand, as the element size is refined, the cohesive surface-induced stiffness reduction will become increasingly significant, which will seriously affect the solution. So, another question arises: what element size should be adopted when using intrinsic CFEM? To answer this question, Tomar et al. [4] have suggested a lower and upper bound of the element size. If the element size is within the two bounds, the crack tip field can be simulated with reasonable accuracy and the influence of the induced stiffness reduction is acceptable. In addition to the element size, the stiffness reduction problem also can be remedied by setting an appropriate stiffness ratio of the cohesive surface to bulk element as suggested in [5–7]. Though these methods can relieve the stiffness reduction effect to a certain extent, the stiffness reduction problem of

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http://dx.doi.org/10.1016/j.engfracmech.2015.01.030
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Nomenclature

A	element area
c_R	Rayleigh wave speed
$\mathbf{C}$	damp matrix
$\tilde{\mathbf{C}}$	transformed damp matrix
d	node number at a spot
E	Young's modulus
$\mathbf{f}$	external force vector
$\tilde{\mathbf{f}}$	transformed external force vector
$\mathbf{F}$	deformation gradient
k_c	initial stiffness of cohesive surface
k_b	initial stiffness of bulk material
$\mathbf{K}$	stiffness matrix
$\tilde{\mathbf{K}}$	transformed stiffness matrix
m	dimension of problem
$\mathbf{M}$	mass matrix
$\tilde{\mathbf{M}}$	transformed mass matrix
$\mathbf{nip}$	constraint array
$\mathbf{ip}$	constraint array
p	restricted degrees of freedom
q	restricted degrees of freedom
$\mathbf{Q}$	constraint matrix
$\mathbf{x}$	material point vector in current configuration
$\bar{\mathbf{x}}$	material point vector in reference configuration
$\mathbf{s}$	nonsymmetric nominal stress tensor
S_{int}	internal cohesive surface area
S_{ext}	external surface area, respectively
T_n	normal traction applied on the cohesive surface
T_t	shear traction applied on the cohesive surface
$T_n^{\max}$	maximum traction of the cohesive law
$\mathbf{T}$	traction vector on a surface in the reference configuration
$\mathbf{u}$	displacement vector
V	volume
δ_n	scale length corresponding to the normal separation
δ_t	scale length corresponding to the shear separation
Δ	displacement jump across the cohesive surface
Δ_n	normal separation of cohesive surface
Δ_t	shear separation of cohesive surface
ϕ_n	normal separation work
ϕ_t	shear separation works
γ	relaxation coefficient
ν	Poisson's ratio
ρ	material density
$\boldsymbol{\sigma}$	stress tensor
$\dot{\varepsilon}_{yy}$	strain rate

the intrinsic CFEM is not well addressed yet. To avoid the stiffness reduction problem, Camacho and Ortiz [8] and Ortiz and Pandolfi [9] inserted cohesive surfaces into the mesh where the stress exceeds the cohesive strength and developed the *extrinsic* CFEM. The extrinsic CFEM employs the initially stiff cohesive law, which has been extensively applied to fracture simulation, e.g. [10–13]. However, the extrinsic CFEM involves local mesh modification and node duplication, which reduces the computational efficiency. To remedy this drawback, Paulino et al. [14] and Zhang et al. [15] developed a topological data structure for extrinsic CFEM implementation.

Actually, during the implementation of the intrinsic CFEM, only a small percentage of the cohesive surfaces, which are located in the high stress zone, really function as designed. Most cohesive surfaces are 'idle'. These idle cohesive surfaces are not only unnecessary, but play a negative role by seriously reducing the material stiffness. If the separations of these idle cohesive surfaces are constrained by introducing a series of constraint conditions, the stiffness reduction problem of the intrinsic CFEM can be addressed to a large extent. In the high stress zone, where fracture can initiate and propagate, the constrained cohesive surfaces are relaxed to function as designed by removing the constraint conditions. This approach is the