Engineering Fracture Mechanics 136 (2015) 265–278

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

A prediction method of fracture probability for lapped brittle plate

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ARTICLE INFO

Article history:
Received 28 October 2012
Received in revised form 18 July 2014
Accepted 2 February 2015
Available online 11 February 2015

Keywords:
Ceramics
Brittle fracture
Surface flaw
Probabilistics
Loose abrasive lapping

ABSTRACT

Fracture of lapped brittle plate exhibits a random and uncertain property. In this study, based on a finite element division and the fracture mechanics of micro surface cracks, a numerical prediction approach for fracture cumulative distribution function (CDF) for lapped brittle plate under non-uniform stress field is proposed. Applications under different conditions are carried out, and influences of abrasive size and crack density on the proposed prediction method are discussed. The predicted Weibull parameters are in good agreement with experimental results, which establishes validation and potential engineering value of the proposed prediction method.

1. Introduction

In some cases, the fracture probability of lapped brittle plate needs to be studied to determine certain threshold values (such as the critical applied load before fracture occurs). For example, in the machining process of aspherical optical workpiece using “elastic deformation machining method”, the plate-shape workpiece is forced to deform to certain aspherical profiles and then be lapped [1,2]. So, when using this kind of machining method, it is necessary to determine whether an aspherical profile is suitable to be the target machining profile, and the key problem is the fracture probability of brittle plate-shaped workpiece when it is lapped and bearing a load. When a surface made of brittle materials (typically the optical glasses) is lapped by loose abrasives, the condition of surface crack is rather complex: the orientation of the crack is random and the crack depth obeys some kind of distribution. When the lapped plate is under certain load-on case (for example simply supported and bearing a uniform pressure), the stress field in the plate is non-uniform. So the fracture of lapped brittle plate exhibits a statistical property. A sampling test is usually adopted to study the fracture property of lapped brittle specimens. But as the lapping process is time-consuming, it is not convenient to carry out the sampling test. Hence many studies have made great efforts to establish the prediction method of fracture probability for lapped brittle workpiece.

The most commonly used distribution of fracture probability for brittle materials is the Weibull distribution [3]. But it is not convenient to determine the Weibull parameters by sampling test for different specimens, so some researchers have made efforts to expand the Weibull distribution to different situations. Wilshaw [4] introduced the concept of “searched

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http://dx.doi.org/10.1016/j.engfracmech.2015.02.005
0013-7944/

Nomenclature

a	surface crack depth (μm)
ac	critical crack depth (μm)
$\langle a \rangle$	mean value of the surface crack depth (μm)
a_{max}	maximum crack depth (μm)
a_{min}	minimum crack depth (μm)
A	exponent of the CDF of surface crack depth
D	inner diameter of the holder (mm)
D_a	average diameter of abrasives (μm)
E	Young's modulus (GPa)
F	total load in "Ring on ring" test (N)
F_1	probability that one single cell will survive
F_2	probability that all the cells will survive
F_3	probability that at least one cell will fail
h	thickness of the plate (mm)
H_K	knoop hardness (GPa)
$H_{lapping}$	lapping hardness ($Pa^{-5/4}\ m^{-1/4}$)
$(H_{lapping})_f$	lapping hardness for float glass ($Pa^{-5/4}\ m^{-1/4}$)
$(H_{lapping})_z$	Lapping hardness for ZERODUR® glass ($Pa^{-5/4}\ m^{-1/4}$)
k_1	constant of Kachalov expression
k_2	lapping coefficient of different abrasives
$(k_2)_f$	lapping coefficient for float glass
$(k_2)_z$	lapping coefficient for ZERODUR® glass
K_{IC}	fracture toughness for mode I ($MPa\ m^{1/2}$)
l	Side length of single cell (μm)
m	Weibull modulus
P	applied pressure (kPa)
r	parameter of crack density function in Ref. [5]
r_1	radius of the loading ring in "Ring on ring" test (mm)
r_2	radius of the supporting ring in "Ring on ring" test (mm)
r_3	radius of the specimen in "Ring on ring" test (mm)
$R_{a/c}$	ratio between crack depth and half length
W	half side length of single cell (mm)
x_i	coordinate value of X axis at the center of single cell (mm)
y_i	coordinate value of Y axis at the center of single cell (mm)
β	parameter of crack density function in Ref. [5]
θ	angle between the crack orientation and the horizontal line
ν	Poisson's ratio
σ	stress in specimen in "Ring on ring" test (MPa)
σ_t	circumferential stress in elastically deformed plate (MPa)
σ_r	radial stress in elastically deformed plate (MPa)

area", $A(\sigma_n, c)$, which means the area of surface within which $K_I \geqslant K_{IC}$ (for a crack of size c) during loading to σ_n. via the concept of "searched area", Warren [5] regarded the Weibull modulus m as a quantity that may vary with stress, and derived the quantitative expressions for m in different load-on situation. Keith et al. [6] gave out the formula of "effective surface area", A_{eff}, which can be computed for a component with a varying stress field. Based on A_{eff}, if Weibull parameters for component under certain kind of load (for example the "Ring on ring" test) are already known, then the Weibull parameters for components under different load can be calculated.

On the other hand, some studies tried to derive the distribution function of fracture probability from a macroscopic view, which provided another possible way to predict the fracture probability of brittle materials. Based on a simple statistical models of volume element, Wallin [7] derived the cumulative failure probability distribution of specimens with a uniform stress state, and the specific scatter distribution indicate a distinct specimen size (crack front length) effect. Nicholson et al. [8] divided one square plate under uniform uniaxial stress into many small square cells, then based on Sih's mixed-mode fracture model of through-wall cracks [9], relations between extreme value distributions for strength of the plate and the size (crack number) of the plate were studied.