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Stress intensity factor calculation for internal circumferential cracks in functionally graded cylinders using the weight function approach

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A B S T R A C T

In this work, we describe stress intensity factors obtained for functionally graded cylinders with internal circumferential cracks using the weight function method for different combinations of cylinder geometry, crack depth, and material gradation. A modified form of the *J* integral is employed to obtain reference stress intensity factors using finite element analysis results. Furthermore, a fitting technique is employed for calculating stress intensity factors not covered in the finite element modeling matrix. The results predicted by the weight function method along with the proposed fitting technique are in good agreement with the results directly obtained from the finite element analysis.

1. Introduction

The weight function method was first introduced in [1,2] and has been widely used to calculate stress intensity factors (SIFs) in cracked structures for arbitrary crack face loading conditions [3]. The weight function is dependent on the crack geometry, but is independent of the applied load. When the weight function is multiplied by the stress applied to the crack face and is integrated over the crack length, it yields the SIF for the given crack and its loading condition [4]. To date, this method has been applied to different crack geometries with different crack face loading conditions and as a result, many closed form SIF expressions or tabulated SIF results are available in the codes of standards for the structural integrity assessment of cracked structures [5].

Cracks in thin walled pipes and thick walled pressure vessels are threats to the safe operation of these structures. For example, cracks in corrosion and cracks close to a girth weld in a pipe are cases in which regulations require that pipeline operators assess the structural integrity of pipe to ensure the safe operation of the pipeline. In past years, significant research has been performed on the analysis of axial and circumferential surface or embedded cracks in pressurized pipes and vessels. As a result, a multitude of analytical and/or tabulated SIF solutions are available in the literature or codes of standard [6–8]. Most of these solutions were developed for cracks in homogenous cylinders based on a combination of FE method and the weight function method. For example, Grebner [9] performed FE analyses for pipes with circumferential surface cracks under axial tensile loading to obtain the corresponding SIFs. Grebner and Mattheck [10] used the weight function method to obtain SIFs for the same pipe geometry, given in [9], under applied linear and quadratic tensile loads. Closed form SIF solutions were

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Nomenclature

a	crack depth
A	area of domain integral evaluation
$A_j (j = 1, \ldots, 5)$	curve fitting parameters
$\boldsymbol{e_r}, \boldsymbol{e_z}$	unit basis vectors of cylindrical coordinate
E_i, E_o	elastic modules at the inner and outer radius of a hollow cylinder
E_{tip}	elastic modulus of the material at the crack tip location
E^*_{tip}	modified plane stress/strain elastic modulus
$h(x, a)$	weight function
J	energy release rate
K	stress intensity factor
K_I	mode-I stress intensity factor
K_I^N	normalized mode-I stress intensity factor
K_r	reference stress intensity factor
L	cylinder length
$M_j (j = 1, 2, 3)$	weight function coefficients
p	material gradation parameter
$\mathbf{P}$	energy-momentum tensor
$\boldsymbol{q}$	vector representing a smooth function, $\boldsymbol{q} = q(r, z)\boldsymbol{e_r}$, where $q(r, z)$ varies from unity to zero
r, z	radial and longitudinal directions in the cylindrical coordinates
R	crack front distance from the axis of symmetry
R_i, R_o	inner and outer radius of a hollow cylinder
S^+, S^-	upper and lower surfaces of the crack in the integration domain
t	cylinder wall thickness
u_r, u_z	radial and longitudinal components of the displacement field
$u_r(x, a)$	crack face displacement field
W	strain energy density
x	distance measured from the inner radius of the cylinder
x_1, x_2	crack tip coordinates
$Y_j (j = 1, 2, 3)$	normalized reference stress intensity factors
β	crack depth to thickness ratio
γ	inner to outer radius ratio
ζ	material gradation parameter
η	ratio of the elastic moduli at the outer surface to the inner surface of the cylinder
λ	material gradation index
ν_{tip}	Poisson's ratio of the material at the crack tip location
ρ	integration domain radius
σ_0	reference uniform stress applied on the crack faces
$\sigma(x)$	applied stress on the crack faces
$\sigma_r(x)$	reference stress applied on the crack faces

Operators

tr	trace of a second order tensor
∇	gradient operator
$\otimes$	dyadic product

obtained by Zahoor [11] for circumferential and axial cracks in pressurized pipes under remote axial and bending loads for different pipe radius to wall thickness ratios. Using the weight function method, Meshii et al. [12] and Meshii and Watanabe [13–15] developed closed-form SIF expressions for finite length hollow cylinders with circumferential cracks. In these works, shell theory was used as the basis of their weight function approach to investigate the effect of finite length of the cylinder on the SIF results. Jones and Rothwell [16] performed a series of FE simulations for various cylinder geometries and crack depth to wall thickness ratios and compared their finite element SIF results with weight function SIF results for internal circumferential cracks. They considered three different reference load cases in their FE analyses to develop the related weight function expressions. Ghajar and Nabavi [17] used the weight functions proposed by Shen and Glinka [18] and developed closed-form expressions for the SIFs of thick-walled cylinders with internal circumferential cracks under pressure and thermal loads.

In recent years, functionally graded materials (FGMs) have gained considerable attention because of their superior resistance to thermal shock and their ability to eliminate interlaminar and residual stresses. In some applications, functionally graded (FG) cylinders may be exposed to excessive thermal gradients, making it necessary to evaluate the degree of severity