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# Numerical analysis of a new mixed mode I/III fracture test specimen

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ABSTRACT

A new test configuration called the edge notched disc bend specimen is suggested for investigating mixed mode I/III fracture behavior. The proposed specimen is a circular disc containing an inclined edge crack thorough the diameter and is subjected to three point bend loading. Extensive three dimensional finite element analyses were performed for different geometrical parameters and loading conditions to obtain the stress intensity factors and the *T*-stress in the proposed disc bend specimen. The finite element results showed that the suggested specimen is able to introduce full combinations of mixed mode I/III ranging from pure mode I to pure mode III.



## 1. Introduction

Many cracked materials including ceramics, rocks, geo materials, some polymers and etc. exhibit brittle or quasi-brittle fracture in practice when they are subjected to service loads. Since the cracked bodies in general have three dimensional geometries and can be subjected to complex stress states, an arbitrary crack inside these structures may experience mixed mode normal-shear deformations. In the past, a large number of research studies have been performed to investigate the mixed mode fracture behavior of cracked components. The review of literature shows that many of the available papers are related to the combined opening- in plane sliding (i.e. mixed mode I/II) fracture [1–12]. For mixed mode I/II loading several test specimens have been proposed which can provide full combinations of modes I and II loadings [1,4,7,13–17]. In addition, a number of mixed mode I/II theoretical fracture criteria, have been developed to predict the onset of brittle fracture under tensile-shear (mixed mode I/II) loading conditions [18–22]. Mixed mode I/III (i.e. combined opening-tearing) fracture, is also one of the frequent and common modes of failure in cracked bodies. For example, shafts and axels containing a circumferential crack and subjected to combined axial–torsional loads, experience a mixed opening-out of plane sliding (i.e. mixed mode I/III) deformation [23–25]. For another example, the pavement of roads containing top-down cracks and subjected to traffic loads of moving vehicles may experience a combination of opening-out of plane sliding [26]. Therefore, it is necessary to investigate the fracture and crack growth behavior of cracks under mixed mode I/III. However, mixed mode I/III fracture has received little attention in comparison with mixed mode I/II case. Among the available published papers in this field, some researchers have studied mixed mode I/III fracture behavior of metallic materials such as steel, carbon steel, aluminum and Titanium [27–35] and also some types of brittle and quasi brittle materials such as PMMA and glass, ceramics, polymer concrete [22,23,36,37] using rod or rectangular beam or plate shape specimens containing circumferential or straight edge cracks. From the experimental view point, it is required to use appropriate test specimens for providing differ-

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**Nomenclature**

| | |
|---|---|
| $a$ | crack depth |
| $a/B$ | crack depth to thickness ratio |
| $B$ | disc thickness |
| $E$ | elastic modulus |
| ENDB | edge notched disc bend |
| $K_{\mathrm{I}}$ | mode I stress intensity factor |
| $K_{\mathrm{II}}$ | mode II stress intensity factor |
| $K_{\mathrm{III}}$ | mode III stress intensity factor |
| $K_{\mathrm{Im}}$ | mode I stress intensity factor at the mid-section of ENDB specimen |
| $P$ | applied load |
| $r$, $\theta$ | crack tip co-ordinate |
| $R$ | disc radius |
| $2S$ | distance between the bottom supports in the ENDB specimen |
| SIF | stress Intensity Factor |
| $S/R$ | loading span to radius ratio |
| $T$ | $T$-stress |
| $T^*$ | non-dimensional $T$-stress |
| $Y_{\mathrm{I}}$ | mode I geometry factor |
| $Y_{\mathrm{III}}$ | mode III geometry factor |
| $Z$ | parameter showing the location of crack front relative to the mid-section |
| $Z/R$ | normalized $Z$ |

*Greek letters*

| | |
|---|---|
| $\beta$ | crack inclination angle |
| $\beta_{\mathrm{III}}$ | pure mode III inclination angle |
| $\sigma$ | reference stress |
| $\sigma_r$, $\sigma_\theta$, $\sigma_z$, $\sigma_{r\theta}$, $\sigma_{rz}$, $\sigma_{z\theta}$ | stress components in polar system |
| $\nu$ | Poisson's ratio |

ent and full combinations of mixed mode I/III. A suitable test specimen for this purpose should have simple geometry and its loading should also be applied easily with the available machines and testing fixtures. The ability of introducing complete and full combinations of modes I and III is another necessary requirement of a good mixed mode I/III test specimen. However, a review of literature shows that most of the available specimens for mixed mode I/III fracture toughness studies have their own shortcomings and limitations. For example, the three dimensional compact tension-shear specimen suggested by Richard et al. [38] for mixed mode I/II/III fracture experiments needs an especial fixture and difficult loading setup for testing. Other specimens such as the three point bend rectangular beam [36,37], pre-cracked bend beam [39], modified compact tension specimen [34,35,40], inclined chevron notched compact tension [41] cannot be used for introducing the complete mode mixities from pure mode I to pure mode III; especially they fail to provide pure or dominantly mode III deformation. Hence the lack of a suitable or standard test specimen is still one of the concerns and interesting subjects for mixed mode I/III fracture toughness studies of different engineering materials.

In this research, a new test configuration called the edge notched disc bend (ENDB) specimen is proposed for mixed mode I/III fracture investigations. It is shown that the suggested specimen can provide complete ranges of mode I/III mixities and hence it can be employed as favorite candidate specimens for mixed mode I/III fracture toughness studies. The fracture parameters of the ENDB specimen including the stress intensity factors and the $T$-stress are computed numerically in the next sections for different crack depths and various combinations of modes I and III.

## 2. The ENDB specimen for mixed mode I/III fracture studies

Fig. 1 shows schematically the geometry and loading configuration of the proposed mixed mode I/III specimen. As shown in Fig. 1 the edge notched disc bend (ENDB) specimen is a circular disc of radius $R$ and height $B$, that contains a straight edge crack of depth $a$ thorough the disc side. The specimen is loaded by three-point bend fixture with span of $2S$. The crack in this specimen makes an angle $\beta$ with respect to the loading supports (see Fig. 1b). When $\beta$ is zero, the specimen is subjected to pure mode I deformation due to symmetry of geometry and loading configuration. But for the other angles of $\beta$ where both geometry and loading is not symmetric relative to the crack plane, the ENDB specimen is subjected to a mixed opening – out of plane sliding (i.e. mixed mode I/III). The contribution of each modes I and III depends on $\beta$ as the main affecting parameter.

The symmetric ENDB specimen (with $\beta = 0^{\circ}$) has been used earlier by Tutluoglu and Keles [42] for determining only the pure mode I fracture toughness of rocks. But in this research we extended its application for mixed mode I/III loading as well