Engineering Fracture Mechanics 134 (2015) 391–403

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Coupled fracture mode of a cracked plate under anti-plane loading

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ARTICLE INFO

Article history:
Received 28 June 2014
Received in revised form 20 December 2014
Accepted 29 December 2014
Available online 7 January 2015

Keywords:
Fracture mechanics
Finite element analysis
Anti-plane loading
Stress intensity factor
Corner point effects

ABSTRACT

The existence of three-dimensional effects at cracks has been known for many years, but understanding has been limited, and for some situations still is. Despite increased understanding, three-dimensional effects are sometimes ignored in situations where they may be important. The purpose of the present investigation is to study a coupled fracture mode generated by a nominal anti-plane (Mode III) loading applied to linear elastic plates weakened by a straight through-the-thickness crack. With this aim accurate 3D finite element (FE) analyses have been performed. The results obtained from the highly accurate finite element models have improved understanding of the behaviour of through cracked plates under anti-plane loading. The influence of plate bending is increasingly important as plate thickness decreases. It appears that a new field parameter, probably a singularity, is needed to describe the stresses at the plate surfaces. Discussion on whether K_{III} tends to zero or infinity as a corner point is approached is futile because K_{III} is meaningless at a corner point. Calculation of the strain energy density (SED) in a control volume at the crack tip allows us to predict the most critical point through the plate thickness.

1. Introduction

Displacements are of fundamental importance in the understanding of the mechanics of materials in general [1–4]. In fracture mechanics the interest is in what happens in the vicinity of the crack tip, so it is sometimes referred to as crack tip surface displacement [1]. Notation used is shown in Fig. 1. If a load is applied to a cracked body, then the crack surfaces move relative to each other. For points on opposing crack surfaces that were initially in contact there are three possible modes of crack surface displacement (mode I, mode II, mode III). By superimposing the three modes, it is possible to describe the most general case of crack tip surface displacement.

If a crack surface is considered as consisting of points then the three modes of crack surface displacement provide an adequate description of the movements of crack surfaces when a load is applied. However, if the surface is regarded as consisting of infinitesimal elements, then element rotations must also be described, and Volterra distorsioni (distortions) are appropriate [5,6]. The crack surfaces may be moved relative to each other in 6 different ways, so there are 6 distinct Volterra distorsioni. These are summarised in Table 1. Elements A and B are on opposite surfaces of a crack and are connected by a ring element around the crack tip (Fig. 2). Modes I, II and III Volterra dislocations correspond to modes I, II and III crack tip surface displacements. The three Volterra disclinations are relative crack surface rotations.

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http://dx.doi.org/10.1016/j.engfracmech.2014.12.021
0013-7944/

Nomenclature

a	crack length
c	as superscript, coupled mode
h	control volume height
K	stress intensity factor, subscripts I, II, III denote mode, superscript c indicates a coupled mode
K_λ	stress intensity measure
p	a given constant
R_0	control volume radius
r	radius of K-dominated region
r, θ	polar coordinates
r, θ, ϕ	spherical coordinates
s	distance from surface
t	thickness
U	displacement subscripts x, y, z denote direction
x, y, z	cartesian coordinates
β	crack surface intersection angle
γ	crack front intersection angle
λ	parameter defining stress intensity measure
ν	Poisson's ratio
ρ	profile tip radius
σ	stress, subscripts x, y, z denote direction
τ	shear stress, subscripts xy, yz, xz denote direction

Crack tip surface displacements in the vicinity of a corner point in which a crack front intersects a surface are often of practical interest. Assuming that Poisson's ratio, $\upsilon > 0$, for the special case in which the crack surface intersection angle, γ (Fig. 3) and the crack front intersection angle, β (Fig. 4) are both 90° then kinematics considerations for an antisymmetric loading [7–10] show that modes II and III crack tip surface displacements cannot exist in isolation [11,12]. Mode II induces mode III^c and mode III induces mode II^c. These induced modes are sometimes called coupled modes, indicated by the superscript c.

Within the framework of linear elastic fracture mechanics [1] the stress field in the vicinity of a crack tip is dominated by the leading term of a series expansion of the stress field [13]. This leading term is the stress intensity factor, K. A particular type of elastic crack tip stress field is associated with each mode of crack tip surface displacement [14] and subscripts I, II and III are used to denote mode. Individual stress components are proportional to $K/\sqrt{r}$ where r is the distance from the crack tip (Fig. 1). Displacements are proportional to $K\sqrt{r}$. A stress intensity factor provides a reasonable description of the crack tip stress field in a K-dominated region at the crack tip, radius $r \approx a/10$ where a is the crack length [1], as shown in Fig. 5.

Simplifying assumptions have become conventional in much present day linear elastic fracture mechanics, for example see Ref. [2], and these are satisfactory for many purposes. The material is assumed to be a homogeneous isotropic continuum,

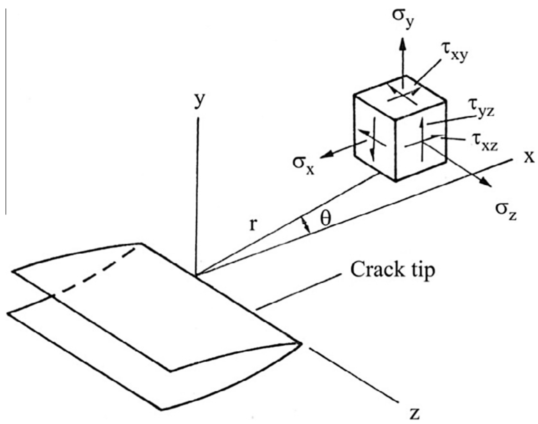

Fig. 1. Notation for crack tip stress field.