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Row of shear cracks moving in one-dimensional hexagonal quasicrystalline materials

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ABSTRACT

Representations for the stress fields created around an infinite row of collinear, antiplane shear cracks moving within one-dimensional hexagonal quasicrystals, and the resulting stress intensity factors and the *J*-integral, are determined in closed-form and discussed, using an extended method of dislocation layers. The solutions for a finite quasicrystalline plate containing a single moving crack and a plate with a moving edge crack are also provided by this analysis.

1. Introduction

Within linear isotropic, and more generally anisotropic, elastic solids, the deformation fields around cracks and inclusions have been analyzed comprehensively using integral transform techniques and complex potential function theory. But it is also well-established that the behaviour of slit-like cracks can be reproduced most conveniently by adopting the techniques of continuous distributions of dislocations. This has been adequately described and applied for isotropic materials by, for example, Bilby and Eshelby [1] and Lardner [2]. During an investigation of cracks in infinite anisotropic solids, the dislocation layer technique was observed by Barnett and Asaro [3] "to be more straightforward and to facilitate computational convenience" more easily than the methods of complex variables/integral transforms.

However, the discovery of quasicrystals was first reported by Shechtman et al. [4], in 1984, and subsequently numerous exciting engineering applications of them have been developed. Thus there has been an increasing interest in studying the behaviour of any embedded cracks, since these are known to significantly affect their mechanical properties and fracture theory. The formulations of the physical and mathematical theories of such materials are well-documented by, for example, Ding et al. [5], Fan [6] and Fan [7].

Various recent studies have been devoted to stationary cracks in one-dimensional hexagonal quasicrystals. Li and Fan [8] derived representations for the phonon and phason stress intensity factors at the tips of two semi-infinite cracks using conformal transformations and complex variable methods. Similar techniques enabled Guo and Liu [9] to analyse the anti-plane shear problem of an elliptic hole with asymmetric collinear cracks. Shi [10] reduced the antiplane sliding mode models of collinear periodic cracks and/or rigid line inclusions to solvable Riemann–Hilbert problems by introducing harmonic functions. The fracture mechanics of four cracks originating from an elliptical hole was studied by Guo and Lu [11], by developing a Stroh-type formulation and then reducing the boundary value problem to Cauchy integral equations using a new mapping function. Then most recently, Guo et al. [12] provided solutions for a crack in a quasicrystal strip using Fourier transforms,

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Nomenclature

b, d	discontinuities in phonon and phason displacements
c	half crack length
c_{ij}	phonon elastic constants
$f(\xi)$, $g(\xi)$	phonon and phason dislocation densities
h	half distance between crack centres
H_{zi}	phason stress components
I_t	regions of cracks
K_1, $K_2(=K)$	phason elastic constants
$K_{\mathcal{T}}$, $K_{\mathcal{H}}$	phonon and phason stress intensity factors
R_1, R_2, R_3 $(=R)$	phonon–phason coupling elastic constants
s_i	wave speeds
t	time
$\mathcal{T}$, $\mathcal{H}$	prescribed phonon and phason stress components
W	strain energy density
u_z, w_z	phonon and phason displacement components
ε_{ij}, w_{zi}	phonon and phason strain components
α, β_i, ε_i	combinations of elastic constants
v	speed of cracks
ξ	moving coordinate
ρ	density
σ_{ij}	phonon stress components

dual integral equations and complete elliptic integrals, and Guo et al. [13] used Westerggaard's stress functions in proposing a semi-inverse method for examining a Griffith crack in an infinite material. These papers also conveniently provide extensive additional references to the earlier literature.

Within an isotropic elastic medium, the displacement and stress fields created around an infinite row of collinear, loaded Griffith cracks has been studied using the general complex variable techniques of Muskhelishvili [14], and others. Koiter [15] applied these to develop a solution for equally-spaced, loaded cracks having equal lengths, during a research project involving "the predominant effect of shear deformations in box beams". Subsequently, England and Green [16] presented an analysis of the case in which each crack is opened by equal and opposite normal pressures that utilized a rather different approach. This involved the adoption of integral representations of the complex potentials which lead to solvable integral equations.

The objective of the current paper is to demonstrate that the components of the fields created around a row of collinear Yoffe cracks moving within one-dimensional hexagonal quasicrystals can be studied particularly conveniently by appropriately extending the dislocation layer technique. No analysis of this physically important situation has been presented previously by any technique. The results of Fan et al. [17], in a study of a dislocation moving uniformly through such materials, provide the foundations of the present study.

In Section 2, the basic theoretical and physical formulation of the situation under consideration is described, together with the appropriate constitutive equations. The required components of the phonon and phason displacement and the corresponding stress fields around a moving quasicrystal screw dislocation are then presented in Section 3. This enables an analysis and derivation to be given in Section 4 of the resulting fields created by the infinite row of collinear, moving cracks and the corresponding stress intensity factors and J-integral. Simultaneously, as discussed in Section 5, this analysis also provides the solutions to the problems of a single moving central crack within a finite quasicrystalline plate and a finite plate having a moving edge crack.

2. Basic theoretical and physical formulation

An infinite periodic array of equal, plane, collinear, Griffith-type, moving, strip cracks of constant width $2c$ which is contained within an infinite homogeneous one-dimensional hexagonal quasicrystal with point group 6 mm is considered. The material is supposed to be everywhere at rest and stress-free in a natural reference state initially, with a uniform density, ρ, and situated so that it is periodic in the $x-y$ plane and quasiperiodic in the positive direction of the z-axis, relative to a fixed system of rectangular Cartesian coordinates (x, y, z).

The constitutive equations relating the components σ_{ij}, H_{zi}, ε_{ij} and w_{zi}, where $i, j = x, y$ or z, of the phonon stress, the phason stress, the phonon strain and the phason strain, respectively, within the quasicrystal are related by constitutive equations, which in matrix notation can be written in the form