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Technical Note

Approximate solution for crack interacting with a gas-filled void

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ABSTRACT

Approximate analytical solution for crack interacting with a gas-filled void is developed on the basis of Eshelby equivalent inclusion theory and transformation toughening theory. As validated by detailed finite element analysis the approximate solution has good accuracy for prediction of mode I, mode II and I/II mixed mode crack-tip stress intensity factors under plane strain loading conditions.

1. Introduction

Gas-filled void (GFV) may be produced in the metal casting process and nucleated and grow in neutron irradiated fusion materials [1]. The GFVs often act as sources of stress concentration, leading to crack nucleation and growth. On the other hand, the gas pressure in the GFV affects growth (or shrinkage) and motion of the GFV [1,2], and thus greatly influences the fracture strength of metals containing GFVs. Therefore, it is of significant to analyze the interaction between crack and GFV. However, due to complexity of the boundary-value problems of elasticity, an analytical solution has not yet been obtained.

Nevertheless, a GFV may be viewed as an inhomogeneous inclusion embedded in matrix [3]. According to Eshelby equivalent inclusion theory [4], an inhomogeneous inclusion can be transformed to a homogeneous one with a transformation strain. Thus, the crack–GFV interaction can then be determined on the basis of the transformation toughening theory [5–8]. Eshelby equivalent inclusion theory and transformation toughening theory provide an alternative approach to solve the problem. Based on this approach we have obtained several approximate analytical solutions for mode I and mode II cracks interacting with an inhomogeneity of arbitrary shape and elastic properties [7–12].

In the present study, a GFV will be converted to a homogeneous inclusion with transformation strain according to Eshelby equivalent inclusion theory. On the basis of transformation toughening theory, some simple approximate formulas in closed-form are developed to predict the influence of the GFV on the crack-tip stress intensity factor (SIF). Detailed finite element analysis is performed to validate the developed formulas.

2. Mode and formulation

Fig. 1a shows the problem to be solved, where a GFV of arbitrary shape is located in crack-tip stress field. The crack-tip SIF will be changed due to presence of the GFV. The crack-tip SIF is a function of the size, shape and location of the GFV, as well

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Nomenclature

a	crack length
$\mathbf{C_m}$	elastic tensor of matrix material
E, ν	Young's module and Poisson's ratio of matrix material
$\mathbf{e}^{\mathrm{A}}$	applied strain tensor in the absence of GFV
$\mathbf{e^T}$	transformation strain tensor
$\mathbf{e}^{\mathrm{I}}$	elastic strain tensor in an equivalent homogenous inclusion
GFV	Gas-filled void
$\mathbf{I}$	unit tensor
K_I^{tip}, K_{II}^{tip}	crack-tip SIFs for mode I and II cracks
K_I, K_{II}	externally applied SIFs for mode I and II cracks
p	gas pressure
R	radius of circular GFV
(r_0, θ_0)	center coordinate of circular GFV
(r, θ)	polar coordinates defined at crack tip
$\mathbf{S}$	Eshelby tensor
SIF	stress intensity factor
Ω	domain occupied by GFV

as gas pressure in the GFV. Evidently, an exact solution for the problem cannot be obtained for a three-dimensional GFV with arbitrary shape. Hence, we simplify the analysis in a plane strain model, which conveys the essence of a three dimensional problem [13]. According to Eshelby equivalent inclusion theory the GFV can be identified to an equivalent homogeneous inclusion with a transformation strain, as shown in Fig. 1b. The stress in the equivalent homogeneous inclusion can be expressed by either

$$\boldsymbol{\sigma} = \mathbf{C}_m \mathbf{e}^{\mathrm{I}}, \tag{1}$$

or

$$\boldsymbol{\sigma} = \mathbf{C}_m \left[(\mathbf{S} - \mathbf{I})\mathbf{e}^{\mathrm{T}} + \mathbf{e}^{\mathrm{A}} \right], \tag{2}$$

where $\mathbf{e}^{\mathrm{I}}$ is the elastic strain in the equivalent homogeneous inclusion, $\mathbf{C}_m$ is the elastic tensor of the matrix material, $\mathbf{S}$ is the Eshelby tensor, depending solely upon the shape of the inclusion and Poisson's ratio of the matrix material, $\mathbf{I}$ is the unit tensor, $\mathbf{e}^{\mathrm{T}}$ is the transformation strain, $\mathbf{e}^{\mathrm{A}}$ is the crack-tip strain in the absence of the GFV. Substituting Eq. (1) into Eq. (2) yields

$$\mathbf{e^T} = (\mathbf{S} - \mathbf{I})^{-1} \boldsymbol{\varepsilon}, \tag{3}$$

where

$$\boldsymbol{\varepsilon} = \mathbf{e}^{\mathrm{I}} - \mathbf{e}^{\mathrm{A}}. \tag{4}$$

Thus, the transformation strain can be determined as long as the elastic strains in the domain Ω occupied by the inclusion are available.

For a differential element $d\Omega$ with circular section in domain Ω (Fig. 1b), the Eshelby tensor for isotropic materials under plane strain condition can be represented by a 3×3 matrix

$$\mathbf{S} = \frac{1}{8(1-\nu)} \begin{bmatrix} 5-4\nu & 4\nu-1 & 0 \\ 4\nu-1 & 5-4\nu & 0 \\ 0 & 0 & 6-8\nu \\ & & \end{bmatrix}, \tag{5}$$

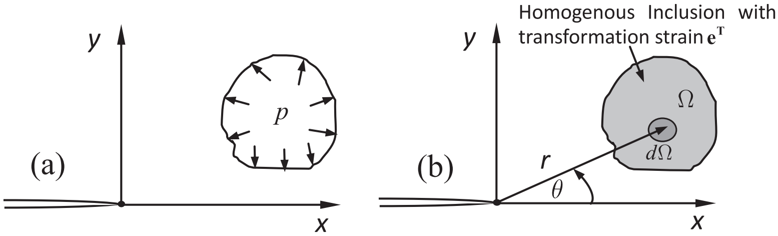

Fig. 1. (a) Crack interacting with a GFV; and (b) the GFV may be equivalent to a homogenous inclusion with transformation strain $\mathbf{e^T}$.