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Transient thermal fracture and crack growth behavior in brittle media based on non-Fourier heat conduction

D.M. Chang, B.L. Wang *

Graduate School at Shenzhen and Center for Composite Materials, Harbin Institute of Technology, Harbin 150001, PR China

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ABSTRACT

In this paper, transient thermal fracture problem corresponding to non-Fourier heat conduction theory in a semi-infinite medium with a surface crack is studied. By Laplace transform and Laplace inverse transform, the analytical solution of the temperature field is obtained. The thermal stress on the crack surface is performed by virtue of the temperature field obtained and superposition method. The thermal stress intensity factor is given in numerical integration form. Crack propagation behavior is discussed and comparison of the results from the Fourier and non-Fourier heat conduction models is also made. It is found that the non-Fourier model is safer than the Fourier model in predicting the thermal shock fracture strength of brittle media.

1. Introduction

The classical Fourier heat conduction law assumes thermal propagation in an infinite speed, which is unreasonable in physical significance to explain some phenomena such as pulsed laser heating [1–3], low temperatures or muscle heat transfer in biology [4,5]. To better explain heat spread in solids, hyperbolic heat conduction equation as one of the non-Fourier theories was developed and applied in short pulse load interval [6,7]. Physically, that supposes a certain time lag for heat flux behind the temperature gradient and the thermal relaxation time was introduced that the temperature field needs to adjust itself to the thermal equilibrium state. Cattaneo [8] and Vernotte [9] first presented the formulation of the non-Fourier heat conduction law, who presented a constitutive equation that was coupled with the local energy balance. The well-known Cattaneo–Vernotte's constitutive equation is

$$\mathbf{q} + \tau \frac{\partial \mathbf{q}}{\partial t} = -k \nabla T \tag{1}$$

where $\mathbf{q} = \mathbf{q}(t + \tau)$ is heat flux vector and T is temperature. The thermal relaxation time τ can be calculated using the thermal wave speed C and the thermal diffusivity $k/(\rho c)$ such as $\tau oc \frac{k}{\rho c} \cdot \frac{1}{C^2}$, in which the non-Fourier heat conduction factor τ is thermal relaxation time, k is the thermal conductivity. To investigate such a non-Fourier heat conduction problem, it is principal to solve the hyperbolic heat conduction equation. Considerable efforts have been devoted to solve the equation, which were mainly divided into analytical and numerical methods. In existed numerical studied, the presence of fictitious numerical oscillations became the primary problems encountered [10,11]. Particularly, it was suggested that it is physically unrealistic when sharp propagation fronts and reflective boundaries are involved [12–14].

Analytical solutions for the hyperbolic heat conduction equation have been obtained for a few relatively simple problems [15–19]. The analytical solutions of hyperbolic heat conduction equations were obtained by Laplace transform method [15].

* Corresponding author.
E-mail addresses: wangbl2001@hotmail.com, wangbl@hitsz.edu.cn (B.L. Wang).

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Nomenclature

q	heat flux vector
T	temperature
σ	stress
ε	strain
τ	thermal relaxation time
k	thermal conductivity
ρ	density
c	specific heat
α	thermal expansion coefficient
c_{ij}	elastic constants
λ_{ij}	stress–temperature coefficients
σ_0	$\bar{\lambda}_{33}T_0$
l_0	$\sqrt{\frac{k\tau}{\rho c}}$
H	unit function
K_I	stress intensity factor
K_{Ic}	fracture toughness

Also the analytical expression of the temperature field for a three dimensional problem was obtained by an auxiliary one dimensional problem solution solved by means of Laplace transform method [16]. Temperature distribution of hyperbolic heat conduction equation in space and time are computed in analytical form for laser pulse load [17]. The hyperbolic heat conduction in cylindrical coordinates subjected to heat flux boundary conditions was investigated by analytical and numerical methods. Compared the results from analytical and numerical solutions, it has up to a 40% error in some cases [18]. Considered by the above references, non-Fourier hyperbolic heat conduction problem for homogeneous materials have also been investigated. The non-Fourier heat conduction problem of functionally graded materials (FGM) for hollow cylinders and spheres was analyzed using the Laplace transform and the modified Durbin's numerical inversion method [19].

On the other hand, a lot of researchers devoted to the thermal shock fracture problem of classical Fourier heat conduction due to its practical significance in engineering applications. For example, the effect of the temperature dependence of thermal properties on the thermal shock fracture tests of ceramics was investigated [20]. The transient thermal stress intensity factor problem for an orthotropic semi-infinite plate and two bonded dissimilar materials were considered in [21,22]. An anisotropic ceramic crystals model was developed to investigate the nonlinear electromechanical behavior of silicon carbide undergoing high temperature shock load [23]. To the author's knowledge, the thermal shock fracture problem and associated crack propagation problem was not studied for non-Fourier heat conduction.

The current paper studies the analytical solutions of temperature and thermal stress fields associated with hyperbolic heat conduction equation for a semi-infinite brittle medium. The thermal stress intensity factor is obtained by a numerical integration formula. The numerical variations of thermal stress and thermal stress intensity factor with thermal shock time, distance depth below the medium surface and crack length are depicted graphically. The crack propagation behavior was also analyzed. Non-Fourier solutions are compared with the Fourier solutions. The significance of non-Fourier heat conduction theory for studying the thermal shock fracture of brittle materials is justified.

2. Temperature and thermal stress field without crack

Considered is a brittle semi-infinite medium shown in Fig. 1. A linear crack of length d exists on the upper surface of the medium. The medium is initially at an uniform temperature T_0. Its upper surface is suddenly cooled to the room temperature at $t = 0$ (here the room temperature is assumed to be zero), which represents the quench experiment for thermal shock. During cooling, the thermal stress becomes tensile on the surface of semi-infinite medium and compressive in the interior of the medium. Therefore, crack may initiate and propagate from the surface towards to the depth direction of the medium. Supposing the surface heat exchange coefficient is infinite, which presents the severest thermal shock to the material. In the

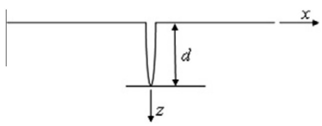

Fig. 1. A semi-infinite medium with a surface linear crack, where d is the crack depth (the medium is initially at an uniform temperature T_0. At $t = 0$, the surface of the medium is suddenly cooled to zero temperature, which is assumed to be zero).