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Three-dimensional stress intensity factors for ring cracks and arrays of coplanar cracks emanating from the inner surface of a spherical pressure vessel

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ABSTRACT

Certain spherical pressure vessels are composed of two hemispheres joined together by a girth weld. These vessels are susceptible to multiple cracking along the weld resulting in one or more cracks developing from the inner surface of the vessel and creating either a ring (circumferential) crack, or an array of coplanar cracks on the equatorial-weld plane. In order to assess the fracture endurance and the fatigue life of such vessels it is necessary to evaluate the Stress Intensity Factors (SIFs) distribution along the fronts of these cracks. However, to date, only two solutions for the SIF for an internal ring crack as well as two 3-D solutions for a ***single*** internal semi-elliptical crack prevailing in various spherical pressure vessels are available. In the present analysis, mode I SIF distributions for a wide range of ring, lunular, and crescentic cracks are evaluated. The 3-D analysis is performed, via the FE method employing singular elements along the crack front. SIFs for numerous ring cracks of different depths prevailing in thin, moderately thick, and thick spherical vessels are evaluated first. Subsequently, three-dimensional mode I SIF distributions along the crack fronts of a variety of lunular and crescentic crack array configurations are calculated for three spherical vessel geometries, with outer to inner radii ratios of $R_0/R_i = 1.01$, 1.1, and 1.7. SIFs are evaluated for arrays of density $\delta = 0–0.99$; for a wide range of crack-depth to wall-thickness ratios, a/t, from 0.025 to 0.95; and for various lunular and crescentic cracks with ellipticities, i.e., the ratio of crack-depth to semi-length, a/c, from 0.2 to 1.5. The obtained results clearly indicate that the SIFs are considerably affected by the three-dimensionality of the problem and by the following parameters: the crack density of the array-δ, the relative crack depth-a/t, crack ellipticity-a/c, and the geometry of the spherical vessel-η. Furthermore, it is shown that in some cases the commonly accepted approach that the SIF for a ring crack of any given depth is the upper bound to the maximum SIF occurring in an array of coplanar cracks, of the same depth, is not universal.

1. Introduction

Due to their optimal specific strength (strength/weight) and their ease of packaging, spherical pressure vessels are widely used for various functions in the chemical, aeronautical, space, nuclear, and armament industries. Furthermore, whenever ***extremely high pressure*** occurs, such as in high explosion containment tanks or in the apparatus used to manufacture

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Nomenclature

a	crack depth
c	crack half length
K_I	mode I SIF
K_I^{Ring}	mode I SIF for a ring crack
K_{max}	maximum SIF
K_0	normalizing SIF (Eq. (1))
K_{00}	normalizing SIF, $K_{00} = p\sqrt{R_i}$
n	number of cracks in the array
Q	shape factor for lunular or crescentic crack (Eq. (3))
p	internal pressure
R_i	inner radius of the spherical vessel
R_o	outer radius of the spherical vessel
r, θ, φ	spherical coordinates
t	spherical vessel's wall thickness

Greek symbols

δ	crack density defined as $\delta = \beta/\theta$ (see Fig. 2)
σ_{rr}	radial stress component
$\sigma_{\theta\theta}$	hoop stress component
$\sigma_{\varphi\varphi}$	meridional stress component
$\bar{\sigma}_{\theta\theta}$	average hoop stress through the spherical vessel's thickness (Eq. (2))
ψ	parametric angle for lunular and crescentic cracks (Fig. 2)

artificial diamonds and other crystals, spherical pressure vessels is the only feasible solution. Some of these spherical pressure vessels are composed of two hemispheres manufactured by: press forming, direct machining, machining of forgings, or by spin-forming. The two hemispheres are joined together by conventional, TIG (Tungsten inert gas), or EB (electron beam) girth weld. These pressure vessels are susceptible to cracking along their girth weld due to one or more of the following factors: cyclic pressurization–depressurization, the existence of a heat-affected zone near the welds, tensile residual stresses within this region, and the presence of corrosive agents. As a result, one or more cracks develop from the inner surface of the vessel creating an array of coplanar cracks on the equatorial-weld plane.

In order to assess the static fracture endurance, crack growth rate, and the total fatigue life of such a pressure vessel it is necessary to determine the prevailing mode I stress intensity factor (SIF)-K_I along the fronts of these cracks. Relatively little attention was given to this problem so far. Atsumi and Shindo [1] and the American Petroleum Institute/ASME Fitness for Service standard, API 579-1/ASME [2] provide stress intensity factors for an internal circumferential crack (ring crack) in various thick and thin spherical vessels. Furthermore, API 579-1/ASME [2] and Hakimi et al. [3] present the only available 3-D solutions for a ***single*** internal semi-elliptical crack in a spherical pressure vessel. These two solutions cover quite a limited range of crack geometries.

In the present study, a full three-dimensional analysis of a cracked thick-walled spherical pressure vessel is performed via the finite element method. In the analysis it is assumed that all the cracks are identical, equi-spaced, and coplanar on the equatorial plane. Ring,[3] lunular,[4] and Crescentic[5] cracks are considered (the little empirical evidence available to the authors at present, points to the fact that inner lunular/crescentic cracks develop in spherical pressure vessels, rather than semi-elliptical ones. However, no experimental data is available to corroborate whether these crack geometries are maintained during crack growth).

First, 3-D SIFs for numerous ring cracks of different depths prevailing in thin, moderately thick, and thick spherical vessels are evaluated. Further on, three-dimensional mode I SIF distributions along the crack fronts of a variety of lunular and crescentic crack array configurations are calculated for three spherical vessel geometries, with outer to inner radii ratios of $R_0/R_i = 1.01$, 1.1, and 1.7. SIFs are evaluated for arrays of density $\delta = 0$–0.99 (crack density is defined as the ratio between two angles $\delta = \beta/\theta$, see Fig. 2); for a wide range of crack-depth to wall-thickness ratios, a/t, from 0.025–0.95; and for various lunular and crescentic cracks with ellipticities, i.e., the ratio of crack-depth to semi-length, a/c, from 0.2 to 1.5.Finally, the commonly accepted approach (see for example[2]) that the SIF for a ring crack of any given depth is the upper bound to the maximum SIF occurring in an array of coplanar cracks, of the same depth, will be critically examined.

[3] A ring crack is defined as a planar circumferential crack of constant depth prevailing on the equatorial plane of a spherical pressure vessel (see Fig. 1).

[4] A lunular crack is defined as a planar, part-through crack whose shape is enclosed by two circular arcs one concave and one convex of different radii, which intersect at two points, having an ellipticity of $a/c = 1$ (see Fig. 2).

[5] A crescentic crack is defined as a planar, part-through crack whose shape is enclosed by two intersecting arches, the concave one which is elliptical, and the convex one which is circular, having an ellipticity of $a/c \neq 1$ (see Fig. 2).