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# A tentative application of the energy separation principle to the determination of the fracture resistance ($J_{Ic}$) of rubbers

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ABSTRACT

In this work an innovative fracture mechanics experimental methodology for *J* testing of rubbers was investigated and tentatively applied. This single-specimen approach, based on the energy separation principle, would allow to identify the point of fracture initiation without any visual observation of the test, and to evaluate the material resistance to crack initiation ($J_{Ic}$). For comparison, fracture initiation was detected also by a methodology based on the visual observation of the crack growth by a camera. Although both methodologies evidenced that fracture initiation in rubbers is a progressive process, the energy separation based approach results to be a promising methodology.



## 1. Introduction

Since elastomer-based systems are highly deformable non-linear viscoelastic materials, global approaches based on energy concepts are preferred in order to characterize experimentally their fracture resistance. Several authors [1–12] proposed the *J*-integral parameter for the characterization of fracture toughness of rubbers. The *J*-integral theory was developed by Rice and Rosengren [13] to characterize the stress and strain field at the crack tip in non-linear materials of infinitesimal elasticity, and in spite of the non-elastic behavior of most rubbers, the path independence of *J*-integral could be verified both experimentally [1] and numerically [2]. *J*-integral is also defined as the energy required for a unit area crack advancement: *J* parameter evaluated at fracture initiation ($J_c$) is expected to provide a material property, independent of the geometry of the tested specimen.

From the experimental point of view, by load–displacement data obtained in a single-specimen fracture test and by specimen geometry data, *J* parameter can be evaluated at any stage of the fracture test according to a simple formula presented later. According to this formula, if the point of fracture initiation is known on the loading curve, $J_c$ value can be easily evaluated in correspondence of that point.

The issue tackled in this work is the determination of the point of fracture initiation. For the determination of such a point, a technique has been recently proposed in literature [3,4] which involves the visual observation of the crack tip during the test by a camera. However the need of the use of a camera or any other optical device to monitor the crack tip can limit such an approach for particular testing conditions, such as high loading rates, as well as high or low temperatures, or aggressive environments, which need that testing is performed inside a chamber. With particular reference to these testing conditions, a method which does not need any visual observation of the test would be useful.

Under such a point of view, a procedure based on the energy separation principle, as proposed by Abdelaziz et al. [14], could be a suitable candidate. In the present work an attempt was done to apply this theoretical principle for the

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**Nomenclature**

| | |
|---|---|
| $a$ | specimen crack length (mm) |
| $a_0$ | specimen initial crack length (mm) |
| $a_i$ | specimen initial crack length of the notched specimen (mm) |
| $a_j$ | specimen initial crack length of the reference specimen (mm) |
| $B$ | specimen thickness (mm) |
| CTOD* | Crack Tip Opening Displacement (mm) |
| $F$ | function appearing in Eqs. (2) and (3) |
| $G$ | geometry function |
| $G_1$ | primitive function of G |
| $H$ | strain energy density function |
| $J$ | fracture resistance ($J$ integral) (kJ/m$^2$) |
| $J_0$ | $J$ at CTOD* = 0 mm (kJ/m$^2$) |
| $J_{0.1}$ | $J$ at CTOD* = 0.1 mm (kJ/m$^2$) |
| $J_{I,lim}$ | $J$ at the limit point (kJ/m$^2$) |
| $J_{Ic}$ | $J$ at fracture initiation (kJ/m$^2$) |
| $k$ | function appearing in Eqs. (2) and (3) |
| $L$ | specimen gauge length (mm) |
| $N$ | function appearing in Eqs. (1) and (2) |
| $P$ | load (kN) |
| $P'$ | load (kN) |
| $R_{ij}$ | energy separation parameter |
| $R_{ij,\max}$ | maximum $R_{ij}$ value reached by the $R_{ij}$ curve after $W_0$ = 0.5 kJ/m$^3$, averaged over the data scatter band |
| $u$ | displacement (mm) |
| $U$ | total external work (J) |
| $U^*$ | complementary total external work (J) |
| $U_i^*$ | complementary energy of the notched specimen (J) |
| $U_j^*$ | complementary energy of the reference specimen (J) |
| $W$ | specimen width (mm) |
| $W_0$ | strain energy density far from the crack (kJ/m$^3$) |
| $\Delta a$ | crack advancement (mm) |
| $\eta$ | geometry dependent calibration factor |
| $\sigma$ | nominal stress (MPa) |

identification of the point of fracture initiation during a fracture test on a rubber specimen; for the application of such methodology, the only experimental measurements required are the recording of the load–displacement curves obtained during the tests on only two specimens.

In the present work two methodologies, one based on the energy separation principle and another one on the visual observation of the crack tip during the test, were applied to the same elastomeric systems. The results are here compared and discussed.

## 2. The energy separation criterion

The energy separation principle was inspired by the work of Andrews [15,16] who developed a generalized theory of fracture mechanics. He defined an energy-loss function to take into account for energy dissipation of a crack growth process in non-elastic and non-linear materials. He obtained the following expression of the energy-loss function, which is equivalent to $J$:

$$J = N\left(W_0, \frac{a}{W}, \frac{a}{L}, \ldots\right) \cdot W_0 \cdot a \tag{1}$$

where $N$ is a function taking into account the finite dimensions of the specimen adopted, $W_0$ is the uniform strain energy density far from the crack, $a$, $W$ and $L$ are the specimen crack length, width and gauge length, respectively.

Abdelaziz et al. [14] tried to dissociate non-linear behavior dependence from finite size effects and proposed to express $N$ as a product of other two functions, $k$ and $F$:

$$N\left(W_0, \frac{a}{W}, \frac{a}{L}, \ldots\right) = k(W_0) \cdot F\left(\frac{a}{W}, \frac{a}{L}, \ldots\right) \tag{2}$$