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Nonlocal models with damage-dependent interactions motivated by internal time

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ABSTRACT

The paper addresses open questions concerning isotropic/anisotropic nonlocal models: possible evolution of internal length and special treatments near boundaries and cracks. First, the nonlocal weight is considered as function of information/wave propagation time normalized by an internal time, which leads to localization with a non-spreading damage zone. The limit value of full damage is attained at a single point in 1D. Second, the WKB approximation for wave propagation in 3D damaged media defines interaction distances as solutions of an eikonal equation. This motivates the interpretation that damage, possibly anisotropic, curves the space in which the interaction distances are calculated.

1. Introduction

The effect and the formulation of boundary conditions – such as free edges, notches and initial cracks – remain an open question for nonlocal models. The main drawback of the standard nonlocal (NL) integral theory [43] consists in the nonphysical interaction, through the nonlocal averaging process, of points across a crack or a hole. The definition of natural boundary conditions of vanishing normal strain gradient at a free edge is still under discussion for implicit gradient formulations [1,40]. The continuous nucleation of a crack of zero thickness is not so simple as the thicknesses of the localization band and also of the band in which damage tends to 1 are not zero. They are proportional to the internal length introduced in the nonlocal approach. Local behavior along free edges – i.e. with a vanishing internal length – has been obtained by some authors [45,25,46,4]. The consideration of an internal length evolving with either the damage, the strain or the stress [19,44,50,36,20,51] seems a way to properly bridge Continuum Damage Mechanics and Fracture Mechanics as the internal length may then vanish for large values of damage. Such a feature is nevertheless quite difficult to enforce in nonlocal integral theories, for instance by keeping the nonlocal connectivity matrix symmetric. See [7,5] for the study of the symmetry of the nonlocal weight.

One attempts here to propose a solution – bringing also questions – to these main difficulties, first within the framework of the integral theory. The idea [13,14] is to keep the nonlocal integral averaging process but to quantify the distance between points as an effective distance, i.e. as a distance function for instance of the geometry and the matter encountered between interacting points. One proposes to define such an effective distance with respect to a dynamic process: how

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Nomenclature

$\langle x \rangle = \max(x, 0)$	positive part of scalar x
$\langle \mathbf{T} \rangle$	positive part of symmetric second-order tensor $\mathbf{T}$
$\lVert \mathbf{x} \rVert$	norm of vector $\mathbf{x}$
$\mathbf{X} : \mathbf{Y}$, $\mathbb{X} : \mathbf{Y}$, $\mathbb{X} \vdots \mathbb{Y}$	contracted products $X_{ij}Y_{ij}, X_{ijkl}Y_{kl}, X_{ijkl}Y_{jkl}$
$\mathbf{x} \otimes \mathbf{y}$, $\mathbf{X} \otimes \mathbf{Y}$	tensorial products $x_i y_j$, $X_{ij}Y_{kl}$
$\mathcal{H}$	Heaviside function, $\mathcal{H}(x) = 1$ if $x \geqslant 0, \mathcal{H}(x) = 0$ if $x < 0$
$\mathbf{1}$	second-order unit tensor
$\mathrm{tr}\, \mathbf{T}$	trace T_{kk} of tensor $\mathbf{T}$
$\mathbf{T}^D = \mathbf{T} - \frac{1}{3}\mathrm{tr}\, \mathbf{T}\, \mathbf{1}$	deviatoric part of second-order tensor $\mathbf{T}$
A, a	anisotropic damage parameters
$c_0, \tilde{c}_0$	wave speed and effective wave speed
$D, \mathbf{D}$	scalar damage variable and second-order damage tensor
E, ν	Young's modulus and Poisson's ratio
$\mathbb{E}$	Hooke's tensor
f	damage criterion function
$\mathbf{g}$	metric of differential geometry
$G, \tilde{G}$	shear modulus and effective shear modulus
$K, \tilde{K}$	bulk modulus and effective bulk modulus
k_0	wave number
ℓ	distance
$\tilde{\ell}$	effective distance
l_c	characteristic/internal length
$\tilde{l}_c$	effective characteristic/internal length
L	one half of bar length
R	radius of nonlocal interaction
S	eikonal function
$\mathbf{u}$	displacement vector
$\mathcal{V}, \mathcal{V}^{\mathrm{nl}}$	local and nonlocal variables
α_0, α	nonlocal weight functions
Γ^k_{ij}	Christoffel symbols
$\hat{\varepsilon}$	Mazars equivalent strain
$\hat{\varepsilon}^{nl}$	nonlocal Mazars equivalent strain
$\boldsymbol{\epsilon}$	strain tensor
ϵ_0	damage threshold
ϵ_f	isotropic damage parameter
η	hydrostatic sensitivity parameter
κ	internal variable (maximum equivalent strain)
$\dot{\lambda}$	damage multiplier
ρ	density
$\boldsymbol{\sigma}$	stress tensor
$\tilde{\boldsymbol{\sigma}}$	effective stress tensor
$\boldsymbol{\sigma}^D$	deviatoric stress tensor
τ_c	internal time
$\tau_{x\xi}$	information propagation time
ω	angular frequency

information (wave) propagates between interacting points. This is made through the introduction of an internal time τ_c, constant, instead of an internal length l_c, measured as evolving. Note that in a similar manner local thermal expansions can be used instead of dynamic impulses [49]. Waves propagations give us information on interacting/non interacting points for the definition of the nonlocal averaging. Such a point of view is consistent with the fact that dynamics is important to define a link between a characteristic time and a characteristic length, either when viscosity is introduced [35,2] or when the physical defects obscuration phenomenon encountered in high speed dynamics and multi-fragmentation is taken into account [9]. Some authors even introduce the standard nonlocal theory by comparing the characteristic wavelength of the deformation field to an intrinsic length of the material [41,21], still a dynamics vocabulary.

The question of the numerical implementation with damage/softening models naturally arises and is addressed in 1D in Section 5, in which the properties of the modified (wave) nonlocal approach are pointed out. The one-dimensional bar test studied has shown its usefulness to explore the localization characteristics of damage models with evolving internal length [44,36,42,51]. An important result of the 1D analysis performed with the nonlocal integral theory with internal time will be